EM Dynamics Modeling and CA's

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Existing EM Modelers

- Method of Moments
- Finite Element Method
- Transmission Line Matrix
- Finite Difference Time Domain

Maxwell's Equations [Yee]

- Maxwell's Equations Differential Form

$$\frac{\partial B}{\partial t} + \nabla \times E = 0 \tag{1}$$

$$\frac{\partial D}{\partial t} - \nabla \times H = J \tag{2}$$

$$B = \mu H \tag{3}$$

$$D = \epsilon E \tag{4}$$

Discretizing Maxwell's Equations

$$-\frac{\partial B_x}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z} \tag{5}$$

$$-\frac{\partial B_y}{\partial t} = \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \tag{6}$$

$$\frac{\partial B_z}{\partial t} = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \tag{7}$$

$$\frac{\partial D_x}{\partial t} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - J_x \tag{8}$$

$$\frac{\partial D_y}{\partial t} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - J_y \tag{9}$$

$$\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - J_z \tag{10}$$

Finite Difference Equations

$$\frac{B_x^{n+1/2}(i,j+\frac{1}{2},k+\frac{1}{2}) - B_x^{n-1/2}(i,j+\frac{1}{2},k+\frac{1}{2})}{\Delta t} \tag{11}$$

$$= \frac{E_y^n(i,j+\frac{1}{2},k+1) - E_y^n(i,j+\frac{1}{2},k)}{\Delta z} \tag{12}$$

$$- \frac{E_z^n(i,j+1,k+\frac{1}{2}) - E_z^n(i,j,k+\frac{1}{2})}{\Delta y} \tag{13}$$

Finite Difference Equations

$$\frac{D_x^n(i+\frac{1}{2},j,k) - D_x^{n-1}(i+\frac{1}{2},j,k)}{\Delta t} \tag{14}$$

$$= \frac{H_z^{n-1/2}(i+\frac{1}{2},j+\frac{1}{2},k) - H_z^{n-1/2}(i+\frac{1}{2},j-\frac{1}{2},k)}{\Delta y} \tag{15}$$

$$- \frac{H_y^{n-1/2}(i+\frac{1}{2},j,k+\frac{1}{2}) - H_y^{n-1/2}(i+\frac{1}{2},j,k-\frac{1}{2})}{\Delta z} \tag{16}$$

$$+ J_x^{n-1}(i+\frac{1}{2},j,k) \tag{17}$$

Yee Cell

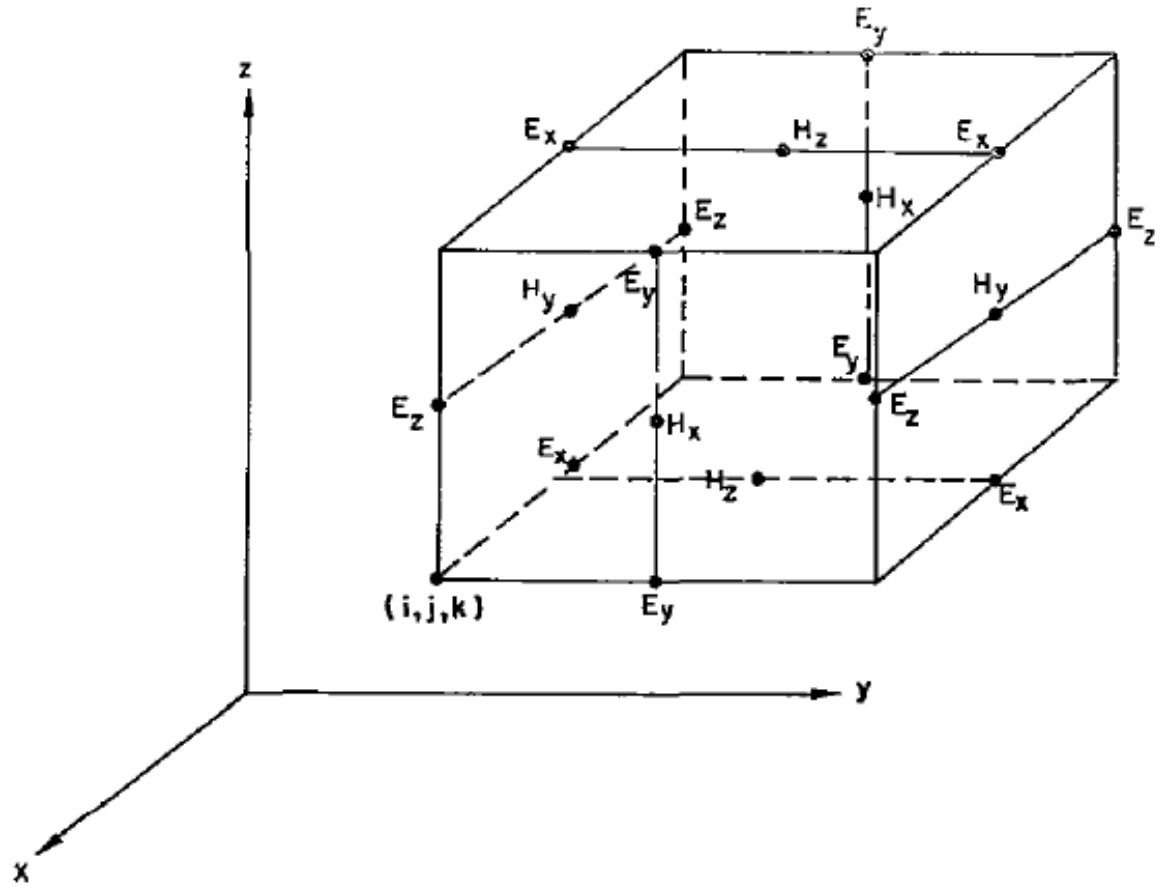
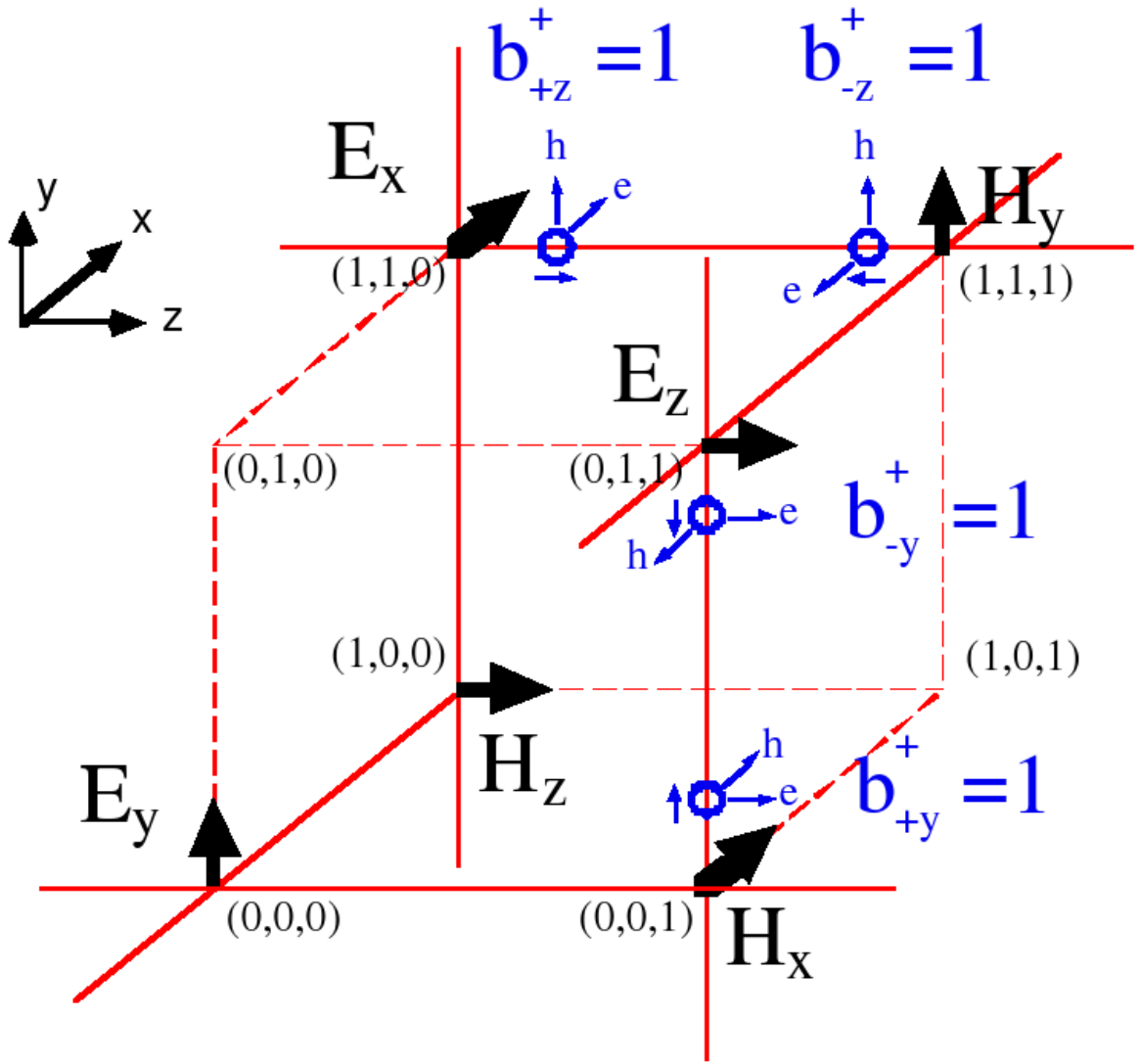


Fig. 1. Positions of various field components. The *E*-components are in the middle of the edges and the *H*-components are in the center of the faces.

Lattice Gas Model 3D



Lattice Gas Model 2D

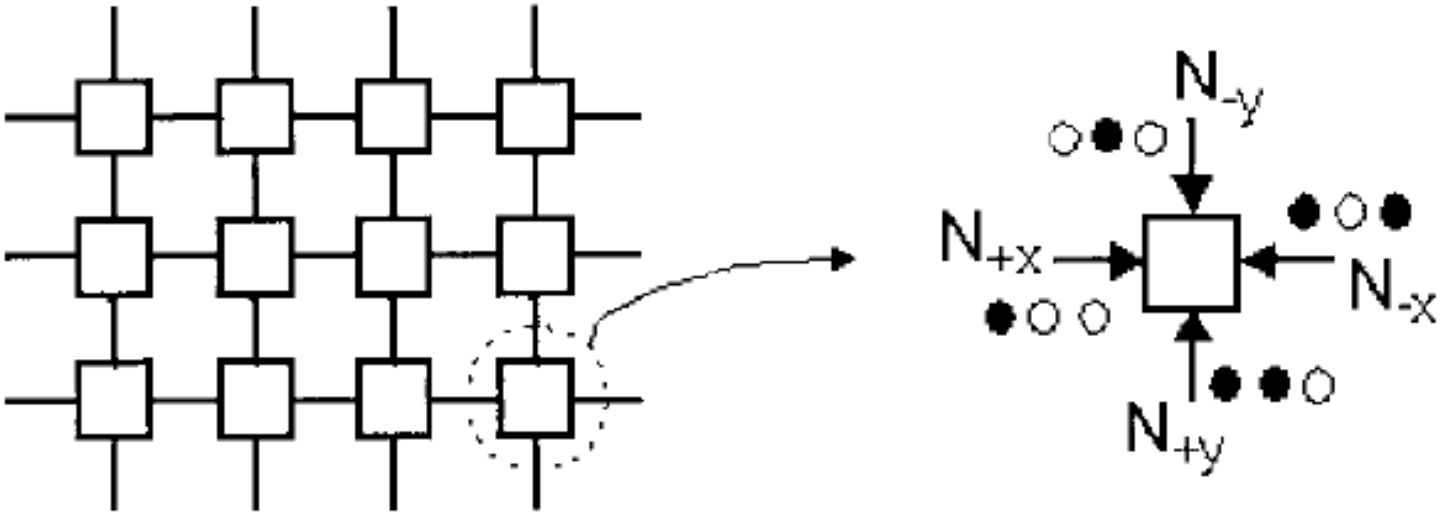


Fig. 1. Lattice gas automaton with low-precision integer variables $N_i \in \{0, 1, \ldots, 2^M - 1\}$ ($M = 3$ bit variables shown).

Guidelines for Implementing CA

- Initial conditions
- Boundary conditions
- Permittivity, permeability, maximum velocity is light speed
- Precision
- Collision operators
- Wave energy and momentum conserved statiscally