(15.1) Do a Taylor expansion of equation (15.6) around V = 0.

$$E \approx 2E_F - 2E_c e^{-2/N_F V}$$

$$E \approx 2E_F - 2E_c \left(\sum_0^{\infty} \frac{f(x)^k}{k!} \right)$$

$$e^{-2/N_F V} = \sum_0^{\infty} \frac{f^n(a)}{n!}(x-a)^n$$

1st $\frac{2e^{-\frac{2}{N_F V}}}{N_F V}$

2nd $\frac{e^{-\frac{2}{N_F V}}(4-4nx)}{N_F^2 V^4}$

3rd $\frac{4e^{-\frac{2}{N_F V}}(3N_F^2 V^2 - 6N_F V + 2)}{N_F^3 V^6}$

The $e^{-\frac{2}{N_F V}}$ term in all of the derivatives dominates and drives all of the derivatives to essentially zero close to the origin. Not sure how to do that assumption "nicely"

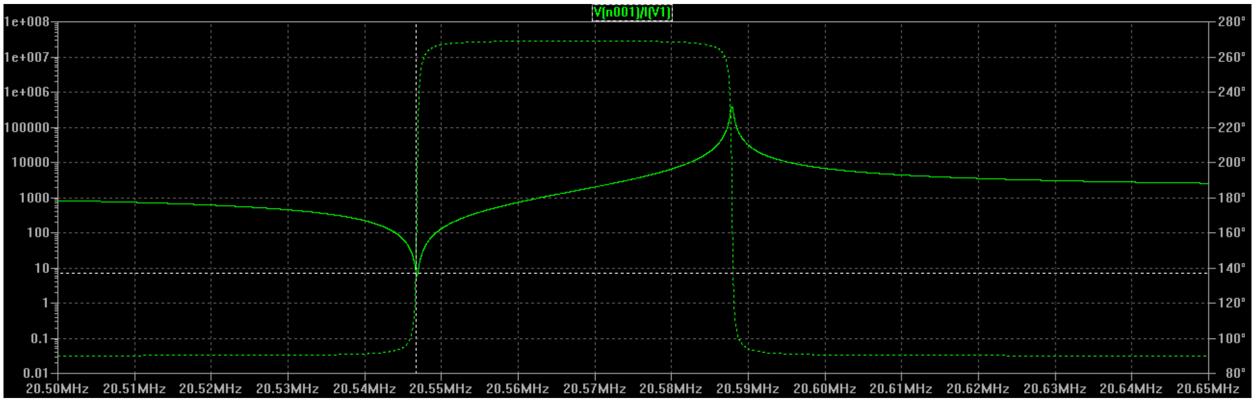
(15.2) If a SQUID with an area of A = 1 cm2 can detect 1 flux quantum, how far away can it sense the field from a wire carrying 1 A?

A flux quantum is $2.07E^{-7}G/cm^2$

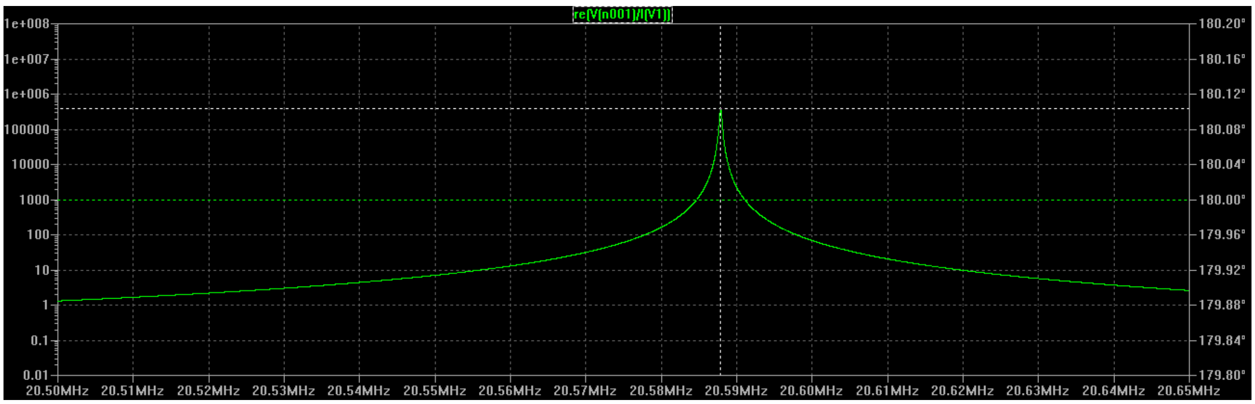
$$\frac{\mu_0 I}{2\pi r} = 2.07E^{-7}\frac{G}{cm^2}(1cm^2)$$

$$r = \frac{\mu_0(1A)}{2\pi}\frac{1}{2.07E^{-7}G} = 9.66km$$

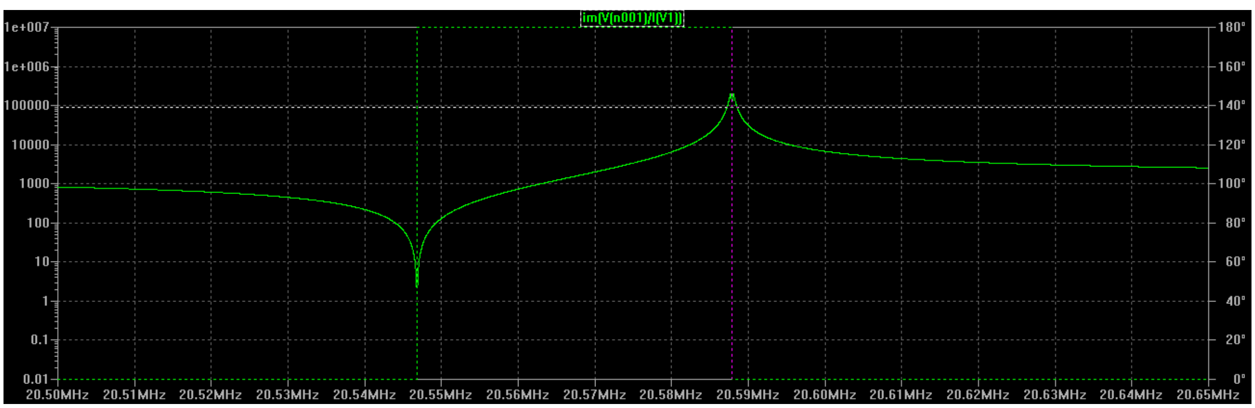
(15.3) Typical parameters for a quartz resonator are Ce = 5 pF, Cm = 20 fF, Lm = 3 mH, Rm = 6 Ω. Plot, and explain, the dependence of the reactance (imaginary part of the impedance), resistance (real part), and the phase angle of the impedance on the frequency.



Phase Magnitude of Impedance



Real component of Impedance



The crystal resonates at $\approx 20.55\mathrm{MHz}$. At this point the imaginary impedance drops to zero and the real impedance remains at 6Ω. This results in a phase of 180 degrees.

(15.4) If a ship traveling on the equator uses one of John Harrison's chronometers to navigate, what is the error in its position after one month? What if it uses a cesium beam atomic clock?

Error is 30 seconds, which directly translates to a longitude error of 30 seconds. This is up to 1km of error (30 * 30.72m/s)

10^-15 * 1 Month = 2.6ns or 81 nm.

(15.5) GPS satellites orbit at an altitude of 20 180 km.

(a) How fast do they travel?

$$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{(6.67E^{-11} m^{3/kg} s\text{^}2)(5.972E^{24} kg)}{(20180km + 6371km)}} = 3.874 km/s$$

(b) What is their orbital period?

$$2\pi r/v = 2\pi(20180km + 6371km)/(3.874km/s\ ) = 11.96\ hr$$

(c) Estimate the special-relativistic correction over one orbit between a clock on a GPS satellite and one on the Earth. Which clock goes slower?

$$\frac{t'}{t} = \frac{1}{\sqrt{1 - v^2/c^2}} = 1 + 8.3E^{-11}$$

Or 83 bonus picoseconds per second

(d) What is the general-relativistic correction over one orbit? Which clock goes slower?

$$t = \frac{1 - \frac{GM}{rc^2}}{1 - \frac{GM}{r'c^2}} t'$$

Or 695 nanoseconds per second

The earth bound clock is slower because it is deeper in the well.