(9.1) Optics (as well as most of physics) can be derived from a global law as well as a local one, in this case Fermat's Principle: a light ray chooses the path between two points that minimizes the time to travel between them. Apply this to two points on either side of a dielectric interface to derive Snell's Law.

Optimize y_1 vs y_2, $Y = y_1 + y_2$

$$t \propto n_1\sqrt{x_1^2 + y_1^2} + n_2\sqrt{x_2^2 + (Y - y_1)^2}$$

Find inflection by taking derivative

$$\frac{d_t}{dy_1} = \frac{n_1 y_1}{\sqrt{x_1^2 + y_1^2}} - \frac{n_2(Y - y_1)}{\sqrt{x_2^2 + (Y - y_1)^2}} = 0$$

$$n_1\frac{y_1}{\sqrt{x_1^2 + y_1^2}} = n_2\frac{y_2}{\sqrt{x_2^2 + y_2^2}}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

(9.2) (a) Use Fresnel's equations and the Poynting vectors to find the reflectivity and transmissivity of a dielectric interface, defined by the ratios of incoming and outgoing energy.

(b) For a glass–air interface (n = 1.5) what is the reflectivity at normal incidence?

(c) What is the Brewster angle?

(d) What is the critical angle?

(9.3) Consider a wave at normal incidence to a dielectric layer with index n2 between layers with indices n1 and n3 (Figure 9.6). (a) What is the reflectivity? Think about matching the boundary conditions, or about the multiple reflections.

(b) Can you find values for n2 and d such that the reflection vanishes?

(9.4) Consider a ray starting with a height r0 and some slope, a distance d1 away from a thin lens with focal length f. Use ray matrices to find the image plane where all rays starting at this point rejoin, and discuss the magnification of the height r0.

[freespace] * [lens] * [freespace] * [start point] = [end point]

$$\begin{bmatrix} 1 & d_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & d_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_0 \\ r_0' \end{bmatrix} = \begin{bmatrix} d_2 \\ d_2' \end{bmatrix}$$

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$$\begin{bmatrix} r_0'\left(d_2 - d_1\left(\frac{d_2}{f} - 1\right)\right) - r_0\left(\frac{d_2}{f} - 1\right) \\ -\frac{r_0}{f} - r_0'\left(\frac{d_1}{f} - 1\right) \end{bmatrix}$$

To make this image, the resulting height needs to be insensitive to r_0':

$$d_2 - d_1\left(\frac{d_2}{f} - 1\right) = 0$$

$$d_2 = d_1\left(\frac{d_2}{f} - 1\right)$$

$$d_2 = \frac{d_1 f}{d_1 - f}$$

$$\frac{1}{d_2} = \frac{1}{d_1} - \frac{1}{f}$$

(9.5) Common CD players use an AlGaAs laser with a 790 nm wavelength. (a) The pits that are read on a CD have a diameter of roughly 1 μm and the optics are diffraction-limited; what is the beam divergence angle?

$$\theta = \frac{\lambda}{\pi n \omega} = \frac{790nm}{\pi(1.00029)1\mu m} \approx .25$$

$$\sin\theta = \frac{\lambda}{2nd} = \frac{790nm}{2(1.00029)1\mu m} \approx 0.39$$

(b) Assuming the same geometry, what wavelength laser would be needed to read 0.1 μm pits?

$$\lambda \propto \omega$$

$$79nm$$

(c) How large must a telescope mirror be if it is to be able to read a car's license plate in visible light (λ ~ 600 nm) from a Low Earth Orbit (LEO) of 200 km?

$$\theta = \frac{600nm}{\pi 10cm} \approx 2E^{-6}$$

$$\theta 200km \approx 0.4m$$