MAS.862 P. I. T.

Problem Set Chapter 11

(11.1) a) We have $\int_{-\varepsilon}^{\varepsilon} E\psi(x)dx = \int_{-\varepsilon}^{\varepsilon} E\left[Ae^{iqx} + Be^{-iqx}\right]dx$

$$= E\left[\frac{A}{iq}e^{iqx} + \frac{B}{-iq}e^{-iqx}\right]_{-\varepsilon}^{\varepsilon} = 0$$

By observing the wanted equation, we have to use that

$$\psi(x) \overset{(11.23)}{=} e^{ikx}u_k(x) \overset{(11.25)}{=} e^{ikx}\left[Ae^{i(q-k)x} + Be^{-i(q-k)x}\right] \quad (1)$$

now we have $\int_{-\varepsilon}^{\varepsilon} \frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}dx = \left[\frac{\hbar^2}{2m}\frac{d\psi(x)}{dx}\right]_{-\varepsilon}^{\varepsilon} =$

$$= \frac{\hbar^2}{2m}\left[\left.\frac{d\psi(x)}{dx}\right|_{x=\varepsilon} - \left.\frac{d\psi(x)}{dx}\right|_{x=-\varepsilon}\right] =$$

$$\overset{(1)}{=} \frac{\hbar^2}{2m}\left[ike^{ik\varepsilon}u(\varepsilon) + e^{ik\varepsilon}\left.\frac{du(x)}{dx}\right|_{x=\varepsilon} - (+ik)e^{+ik(-\varepsilon)}u(-\varepsilon) - e^{-ik\varepsilon}\left.\frac{du(x)}{dx}\right|_{x=-\varepsilon}\right] \quad (2)$$

Taking the $\lim_{\varepsilon\to 0}$ of (2) we have $\frac{\hbar^2}{2m}\left[\cancel{iku(0)} + \left.\frac{du(x)}{dx}\right|_{x=0} - iku(0) - \left.\frac{du(x)}{dx}\right|_{x=\Delta 0}\right]$

$$\text{or } \frac{\hbar^2}{2m}\left[(iq-k)A + (-i(q-k))B - i(q-k)Ae^{i(q-k)\Delta} - (-i(q-k))Be^{-i(q-k)\Delta}\right]$$

$$\text{or } \frac{\hbar^2}{2m}iq\left[A + B - Ae^{i(q-k)\Delta} + Be^{-i(q-k)\Delta}\right] \text{ which is the RHS of 11.28}$$

For the LHS: $\lim_{\varepsilon\to 0}\int_{-\varepsilon}^{\varepsilon}\sum_{n=\infty}^{\infty} V_0\,\delta(x-n\Delta)\psi(x)dx = \begin{cases} 0 & \text{everywhere } x-n\Delta \neq 0 \\ \psi(0) & \text{when } x-n\Delta = 0 \end{cases}$

$$= V_0\psi(0) = V_0(A+B)$$

□

which is the LHS of 11.28

(11.1)(b) From (11.26), we have $A+B = Ae^{i(q-k)\Delta} + Be^{-i(q+k)\Delta} \Rightarrow$

$$\Rightarrow A = \frac{B\left(e^{-i(q+k)\Delta} - 1\right)}{1 - e^{i(q-k)\Delta}} \quad (1)$$

And also from (11.28): $\frac{\hbar^2}{2m} iq\left(A - B - Ae^{i(q-k)\Delta} + Be^{-i(q+k)\Delta}\right) = V_0(A+B)$ (2)

Substituting (1) → (2)

if $\xi = e^{-i(q+k)\Delta}$, $p = e^{i(q+k)\Delta}$ $\Rightarrow \frac{\hbar^2}{2m} iq\left[B\left(\frac{\xi - 1}{1-p}\right) - B - B\frac{\xi-1}{1-p}p + B\xi\right] = V_0 B\left(\frac{\xi-1}{1-p} + 1\right)$

or $\frac{\hbar^2}{2m} iq\left(B\xi - B - B + Bp + B\xi p + Bp + B\xi + B\xi p\right) = V_0\left(B\xi - B + B - pB\right)$

or $\frac{\hbar^2}{2m} iq\, 2\left(B\xi + Bp + B\xi p - B\right) = V_0 B(\xi - p)$

or $\frac{\hbar^2}{m} iq B\left(\xi + p + \xi p - 1\right) = V_0 B(\xi - p)$

Changing back variables and grouping to look like (11.29):

$$\left[ie^{-i(q+k)\Delta} + ie^{i(q+k)\Delta} + i\underbrace{e^{-iq\Delta - ik\Delta + iq\Delta + ik\Delta}}_{1} - i - \frac{mV_0}{q\hbar^2}\left(e^{-i(q+k)\Delta} - e^{i(q+k)\Delta}\right)\right]B = 0$$

Observing that $ie^{-i(q+k)\Delta} + ie^{i(q+k)\Delta} = i\left[\cos(+(q+k)\Delta) - i\sin(+(q+k)\Delta) + \cos((q+k)\Delta) + i\sin((q+k)\Delta)\right]$

$= i\,2\cos((q+k)\Delta)$

likewise $e^{-i(q+k)\Delta} - e^{i(q+k)\Delta} = i\,2\sin((q+k)\Delta)$

Substituting back: $\left[i\,2\cos((q+k)\Delta) - 2i\frac{mV_0}{q\hbar^2}\sin((q+k)\Delta)\right]B = 0$

$$\Rightarrow \left[\cos((q+k)\Delta) - \frac{mV_0}{q\hbar^2}\sin((q+k)\Delta)\right]B = 0$$

(11.2) $f(E) = \frac{1}{1+e^{(E-\mu)/kT}} \overset{11.35}{\simeq} \frac{1}{1+e^{(E-E_F)/kT}}$

For the conduction band edge we want $E = E_c$

So $f(E_c) = \frac{1}{1+e^{(E_c-E_F)/kT}} = \frac{1}{1+e^{\frac{2E_c-E_c+E_v}{2}/kT}} = \frac{1}{1+e^{E_g/2kT}} =$ $\simeq 2.5\times10^{-6}$ Ge, $\simeq 5.4\times10^{-10}$ Si, 1.7×10^{-42} diamond

intrinsic
So $E_F = E_g/2 = \frac{E_c-E_v}{2}$

(11.3) (a) We have $N_n = 2\left(\frac{m_n^* kT}{2\pi\hbar^2}\right)^{3/2} \overset{(Si)}{=} 2\left(\frac{1.1\, m_0 kT}{2\pi\hbar^2}\right)^{3/2}$

$$= 2\cdot\left(\frac{1.1\times 9.11\times10^{-28}\,g\; 1.38\times10^{-23}\,JK^{-1}\times 300\,K}{2\pi\,(1.054\times10^{-34})^2 J^2 s^2}\right)^{3/2}$$

$$= 2\left(\frac{4148.694\times10^{-51}\,g\,J}{6.98\times10^{-68}\,J^2s^2}\right)^{3/2}$$

$$= 2\,(594.36\times10^{-17})^{3/2}\,cm^{-3} \simeq 2.9\times10^{19}\,cm^{-3}$$

Likewise we calculate $N_p \overset{(Si)}{=} 2\left(\frac{0.56\, m_0 kT}{2\pi\hbar^2}\right)^{3/2} = \dots \simeq 1\times10^{19}\,cm^{-3}$

Now we can calculate the intrinsic density

$$n_i^2 = N_n N_p\, e^{-E_g/kT} \overset{(Si)}{=} 2.9\times10^{19}\,cm^{-3}\times10^{19}\,cm^{-3}\cdot e^{\frac{-1.11eV}{0.026eV}}$$

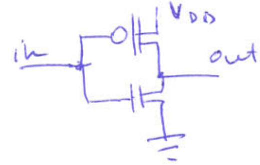
$$= 2.9\times10^{28}\times 2.87\times10^{-19}\,cm^{-6}$$

$$\simeq 8\times10^{19}\,cm^{-6}$$

We know $n_i^2 = n\cdot p \Rightarrow p = \frac{n_i^2}{n} = \frac{8\times10^{19}\,cm^{-6}}{10^{17}\,cm^{-3}} = 800\,cm^{-3}$

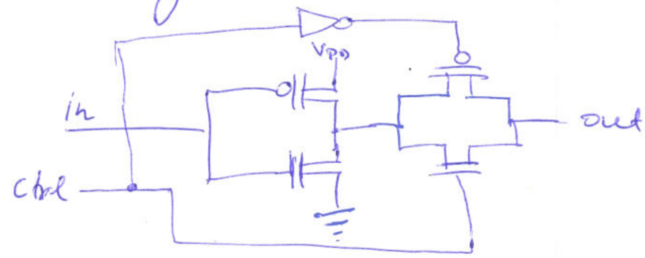
(b) From (11.46) we have $p = n_i\, e^{(E_i-E_F)/kT} \Rightarrow E_i - E_F = \ln\left(\frac{p}{n_i}\right)\cdot kT \simeq 0.4\,eV$

(11.4) Regular CMOS inverter:

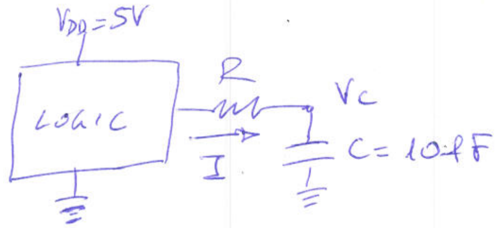


We can add a transmission gate to the output

So we will have



(11.5)



(a) $E_C = \frac{1}{2} C V^2 = \frac{1}{2} 10fF \cdot 5^2 V^2 = 1.25 \times 10^{-13} J$

(b) The current that charged the capacitor had to pass through the resistor so the total dissipated is equal to the total stored.

Proof: $I = \frac{V - V_c}{R} \Rightarrow C \frac{dV_c}{dt} = \frac{V - V_c}{R} \Rightarrow \frac{dV_c}{dt} = \frac{1}{RC}(V - V_c)$

which gives the well known charging curve $V_c = V(1 - e^{-t/RC})$ and $I = \frac{V}{R} e^{-t/RC}$

So $E_R = \int_0^\infty I^2 R \, dt = \int_0^\infty \frac{V^2}{R^2} e^{-\frac{2t}{RC}} R \, dt$

$= \int_0^\infty V^2/R \cdot e^{\frac{-2t}{RC}} dt$

$= -\frac{RC}{2} \frac{V^2}{R} \left[e^{-2t/RC} \right]_0^\infty$

$= \frac{CV^2}{2} = E_C$

(11.5) (c)

(d) We want $P = 1W$

during charging we have $E_R = CV^2/2 = 1.25 \times 10^{-13} J$

during discharging we have again $E_R = 1.25 \times 10^{-13} J$

So total energy dissipation in a cycle is $E = 2E_R = 2.5 \times 10^{-13} J$

So $P = \frac{E}{T} = 1W \Rightarrow f = \frac{1W}{2.5\, 10^{-13} J} = 4 \times 10^{12} Hz = 4\, THz$

(e) $P_{total} = N \cdot E \cdot f = 10^6 \times 2.5 \times 10^{-13} J \times 100 \times 10^6 s^{-1} = 25W$

(f) First lets find the total charge stored:

$$Q = C \cdot V = 10 \times 10^{-15} F \cdot 5^V = 5 \times 10^{-14} C$$

So $n_e = \frac{Q}{q_e} = \frac{5 \times 10^{-14} C}{1.6 \times 10^{-19} C} \simeq 3 \times 10^5$ electrons