

Problem Set Chapter 13

(13.1) (a) From (13.15) $\chi_m = -\mu_0 \frac{q^2 Z r^2}{4 m e V}$

Assuming a single electron orbiting at distance $r = 5.3 \times 10^{-11} \text{ m}$ (Bohr radius)
 Thus the effective volume is $V = \frac{4\pi}{3} r^3 \approx 6.22 \times 10^{-31} \text{ m}^3$

Thus for a hydrogen atom: $\chi_m = -4\pi \times 10^{-7} \frac{(1.602 \times 10^{-19} \text{ C})^2 \times (5.3 \times 10^{-11})^2}{4 \cdot (9.11 \times 10^{-31} \text{ kg}) \times 6.22 \times 10^{-31} \text{ m}^3}$
 $\approx -10^{-5}$

✓ notes

(b) $F = -V \mu_0 \chi_m H \frac{dH}{dz}$

Across the trap drops to zero so: $\frac{dH}{dz} = \frac{H}{r_{\text{mag}}} \approx \frac{H}{0.2 \text{ m}}$
 Assuming g is 0.5 kg then we have:

$0.5 \times 9.8 \text{ kg m/s}^2 = - (0.2 \text{ m})^3 \times 4\pi \times 10^{-7} \frac{\text{H}}{\text{m}} \times 10^{-5} \times \frac{H^2}{0.2 \text{ m}}$
 or $H \approx 10^5 \text{ A/m}$
 $\Rightarrow B \approx 1.25 \text{ T}$

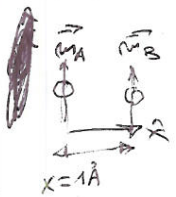
(13.2) $\vec{B} = \frac{\mu_0}{4\pi} \left[\frac{3 \hat{x} (\hat{x} \cdot \vec{m}) - \vec{m}}{|\vec{x}|^3} \right]$

From (13.19) an electron has dipole moment $\mu_B = 9.274 \times 10^{-24} \text{ J/T}$

Assuming they are spaced 1 \AA away with same dipole moment vectors
 we have $U_m = \mu_B \vec{B}$

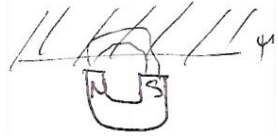
So $U_m = \mu_B \frac{\mu_0}{4\pi} \frac{2\mu_B}{|\vec{x}|^3} = \frac{(9.274 \times 10^{-24} \text{ J/T})^2 \times 4\pi \times 10^{-7} \text{ H m}^{-1}}{4\pi \times (10^{-10} \text{ m})^3} \approx 10^{-4} \text{ eV}$

$U_C = qV_C = \frac{q^2}{4\pi\epsilon_0 r} = \frac{(1.602 \times 10^{-19} \text{ C})^2}{4\pi \times 8.85 \times 10^{-12} \text{ F m}^{-1} \times 10^{-10} \text{ m}} \approx 14 \text{ eV}$



(13.3) Energy in magnetic field: $U = \frac{1}{2} \int \vec{B} \cdot \vec{H} dV = \frac{1}{2} \int \frac{B^2}{\mu} dV$

(a) For ferromagnets we have $\mu \gg 1$



So if the permanent magnet has a field B , the returning energy will be much lower (by a factor of μ). The difference in energy is the reason for the attractive force.

(b) $E = \frac{1}{2} \int \vec{B} \cdot \vec{H} dV = \frac{1}{2} \int \vec{B} (\vec{H}_0 + \vec{M}) dV = \frac{1}{2} \int \frac{B^2}{\mu_0} - \vec{M} \cdot \vec{B} dV$



Magnetic field lines want to go where is lower permeability

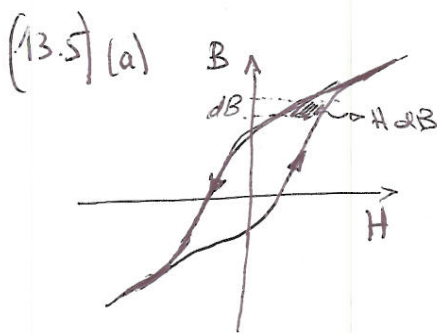
(13.4) From (13.21) we have:

$$M = \frac{m}{V} = \mu_B \cdot \frac{N_e}{V} \approx \mu_B \cdot \frac{\text{atoms}}{V}$$

For Fe_2 : a.w = 55.84
density = 7.874 g/cm³

$$\Rightarrow M = \mu_B \cdot \frac{NA}{a.w} \cdot \text{density} = 9.27 \times 10^{-24} \text{ J/T} \cdot \frac{6.022 \times 10^{23}}{55.85 \text{ g}} \times \frac{7.874 \text{ g}}{1 \text{ cm}^3} \times \frac{1 \text{ cm}^3}{(0.01 \text{ m})^3}$$

$$\Rightarrow M \approx 7.7 \times 10^5 \text{ A/m}$$



$$\frac{\partial U}{\partial t} = \vec{E} \cdot \frac{\partial \vec{D}}{\partial t} + \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}$$

Integrating over the curve and assuming $\frac{\partial B}{\partial t} dt = dB$

$$\int \frac{\partial U}{\partial t} dt = \int \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} dt$$

$$\Rightarrow U = \int \vec{H} \cdot d\vec{B} = \int_{\text{left}} \vec{H} \cdot d\vec{B} - \int_{\text{right}} \vec{H} \cdot d\vec{B}$$

(b)

(A36) ~~Find the magnetic field~~

We know: $H_r = \frac{I}{2\pi r}$

$$\rightarrow I = 2\pi r H = 2\pi \times 1\text{cm} \times 3000 \times \frac{1\text{A m}^{-1}}{4\pi \times 10^{-3}\text{Oe}}$$

$$\rightarrow I = 1500\text{A}$$

