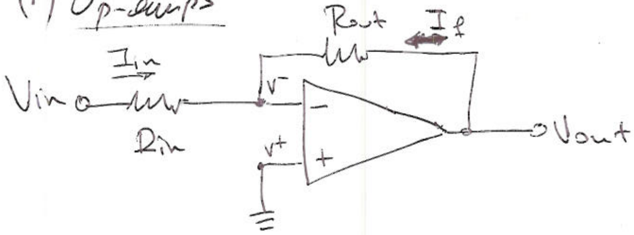
MAS. 862 P.I.T.

Chapter 14 Problem Set

(1) Op-amps



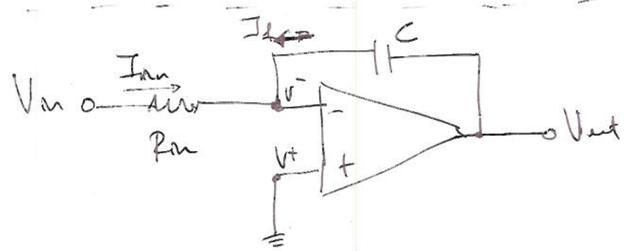
$$I_{in} = -\frac{V^- - V_{in}}{R_{in}}$$

$$I_f = \frac{V_{out} - V^-}{R_{out}}$$

$$V^- = V^+ = 0$$

$$I_{in} = -I_f$$

$$\Rightarrow -\frac{V_{in}}{R_{in}} = \frac{V_{out}}{R_{out}} \Rightarrow \boxed{V_{out} = -\frac{R_{out}}{R_{in}} V_{in}}$$



$$I_{in} = -\frac{V^- - V_{in}}{R_{in}}$$

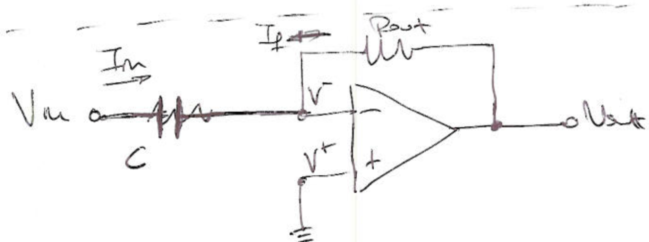
$$I_f = C\frac{d(V_{out} - V^-)}{dt}$$

$$V^- = V^+ = 0$$

$$I_{in} = -I_f$$

$$\Rightarrow \frac{-V_{in}}{R_{in}} = C\frac{dV_{out}}{dt}$$

$$\Rightarrow \boxed{V_{out} = -\frac{1}{R_{in}C}\int_t V_{in}\,dt}$$



$$I_{in} = C\frac{d(V^- - V_{in})}{dt}$$

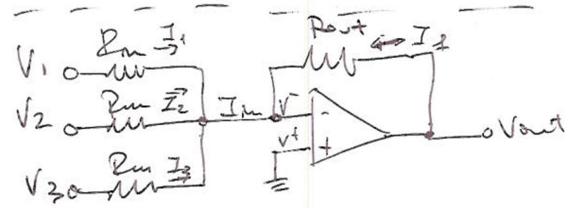
$$I_f = \frac{V_{out} - V^-}{R_{out}}$$

$$V^- = V^+ = 0$$

$$I_f = -I_{in}$$

$$\Rightarrow -C\frac{dV_{in}}{dt} = \frac{V_{out}}{R_{out}} \Rightarrow$$

$$\Rightarrow \boxed{V_{out} = -R_{out}C\frac{dV_{in}}{dt}}$$



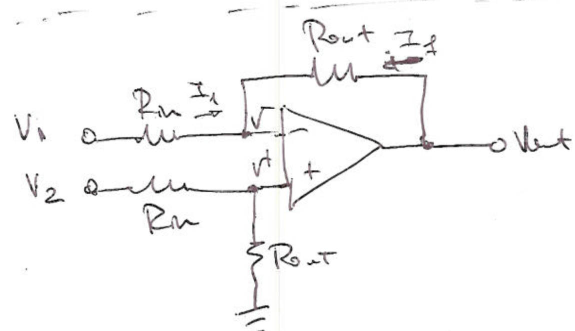
$$I_{in} = I_1 + I_2 + I_3 = \frac{\cancel{V^-} - (V_1 + V_2 + V_3)}{R_{in}}$$

$$I_f = \frac{V_{out} - V^-}{R_{out}}$$

$$V^- = V^+ = 0$$

$$I_f = -I_{in}$$

$$\Rightarrow \boxed{V_{out} = -\frac{R_{out}}{R_{in}}(V_1 + V_2 + V_3)}$$



$$I_1 = -\frac{V^- - V_1}{R_{in}} \quad (1)$$

$$I_f = \frac{V_{out} - V^-}{R_{out}} \quad (2)$$

$$(3) \quad V^- = V^+ = \frac{R_{out}}{R_{in} + R_{out}} \cdot V_2$$

$$(4) \quad I_1 = -I_f$$

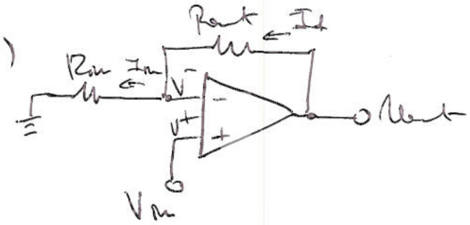
$$\xRightarrow{1,2,4} R_{out}(V^- - V_1) = R_{in}(V_{out} - V^-)$$

$$\Rightarrow (R_{in} + R_{out})V^- = R_{in}V_{out} + R_{out}V_{in}$$

$$\xRightarrow{3} R_{out}V_2 = R_{in}V_{out} + R_{out}V_{in}$$

$$\Rightarrow \boxed{V_{out} = \frac{R_{out}}{R_{in}}(V_2 - V_1)}$$

(1) (b)



$$I_{in} = \frac{V^- - 0}{R_{in}}$$

$$I_f = \frac{V_{out} - V^-}{R_{out}}$$

$$V^- = V^+ = V_{in}$$

$$I_f = I_{in}$$

$$\Rightarrow \frac{V_{in}}{R_{in}} = \frac{V_{out} - V_{in}}{R_{out}}$$

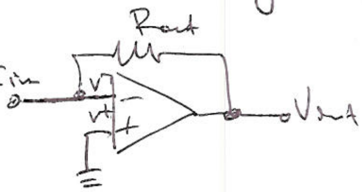
$$\Rightarrow V_{out} = R_{out} \cdot \left(\frac{1}{R_{in}} + \frac{1}{R_{out}}\right) V_{in}$$

$$\Rightarrow \boxed{V_{out} = \left(1 + \frac{R_{out}}{R_{in}}\right) V_{in}}$$

Reasons not used

- Gain > 1 always
- Cannot sum voltages without common-mode rejection (voltage vs current sum)

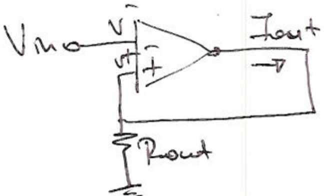
(c)



$$\frac{-V_{out} + V^-}{R_{out}} = I_{in}$$

$$V^- = V^+ = 0$$

$$\Rightarrow \boxed{V_{out} = -R_{out} \, I_{in}}$$

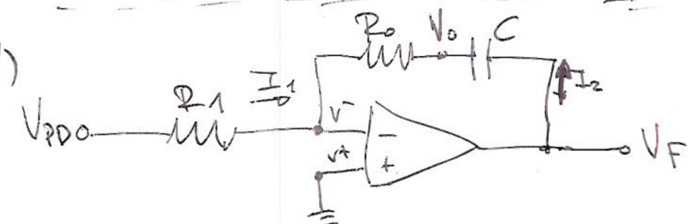


$$V^+ = I_{out} \cdot R_{out}$$

$$V^- = V^+ = V_{in}$$

$$\Rightarrow \boxed{I_{out} = \frac{V_{in}}{R_{out}}}$$

(d)



$$\frac{-V^- + V_{PDO}}{R_1} = I_1 \quad ①$$

$$V^- = V^+ = 0 \quad ④$$

$$\frac{V_0 - V^-}{R_0} = I_2 \quad ②$$

$$I_1 = -I_2 \quad ⑤$$

$$C \frac{d(V_F - V_0)}{dt} = I_2 \quad ③$$

So we have (①, ③, ⑤): $\frac{V_{PDO}}{R_1} = C \frac{d V_F - V_0}{dt} \Rightarrow \frac{V_{PDO}}{R_1} = -C \frac{dV_F}{dt} + C \frac{dV_0}{dt}$

and (①, ②, ⑤): $\frac{V_{PDO}}{R_1} = -\frac{V_0}{R_0} \Rightarrow V_0 = -V_{PDO} \cdot \frac{R_0}{R_1}$

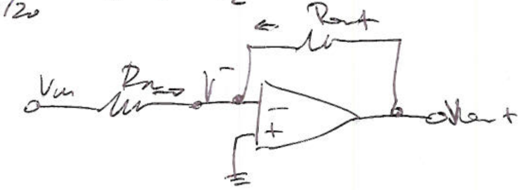
$\Rightarrow$

$$\Rightarrow \frac{V_{PDO}}{R_1} = -C \frac{dV_F}{dt} - C \frac{R_0}{R_1} \frac{dV_{PDO}}{dt}$$

$$\Rightarrow \boxed{\frac{dV_F}{dt} = -\frac{R_0}{R_1} \frac{dV_{PDO}}{dt} - \frac{V_{PDO}}{R_1 C}}$$

(14.2) We have eq(14.3): $\omega_{ol} = \frac{\omega_{cl}}{G_{ol}} = \frac{10^6\,Hz}{10^{100/20}} = 10\,Hz$

For the ~~non~~-inverting amplifier we have



By definition $G_{ol} = -\frac{V_{out}}{V^-}$ (op-amp)

and we have $\frac{V_{in}-V^-}{R_{in}} = -\frac{V_{out}-V^-}{R_{out}} \Rightarrow V^- = \left(\frac{V_{in}}{R_{in}} + \frac{V_{out}}{R_{out}}\right)\cdot\frac{R_{in}+R_{out}}{R_{in}\cdot R_{out}}$

Substituting $-\frac{V_{out}}{G_{ol}} = \left(\frac{V_{in}}{R_{in}} + \frac{V_{out}}{R_{out}}\right)\cdot\frac{R_{in}+R_{out}}{R_{in}\cdot R_{out}} = \frac{(R_{out}V_{in} + R_{in}V_{out})(R_{in}+R_{out})}{(R_{in}\cdot R_{out})^2}$

$\Rightarrow \frac{V_{out}}{V_{in}} = -\frac{G_{ol}\,R_{out}/R_{in}}{G_{ol} + 1 + R_{out}/R_{in}} = \cancel{G_{cl}} - \frac{G_{ol}}{1 + \frac{R_{in}G_{ol}}{R_{out}} + \frac{R_{in}}{R_{out}}}$

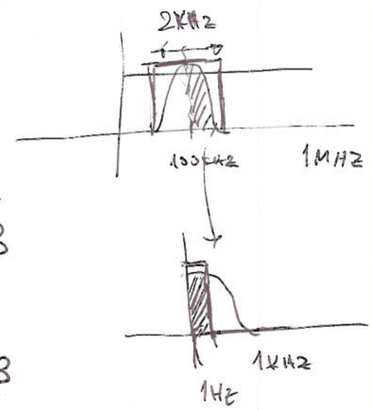
Using (14.2) we have $G_{cl}(\omega) = \frac{G_{ol}}{1 + i\frac{\omega_0}{\omega_{ol}}} = \frac{G_{ol}}{1 + i\,G_{ol}}$

(14.3) $f_{osc} = 100\,kHz$

$Q_{BPF} = 50 \rightsquigarrow$ filter width: 2kHz

$\tau_{LPF} = 1s \rightsquigarrow f_{LPF} = 1\,Hz$

$f_{inp\,max} = 1\,MHz$



$\rightsquigarrow$ noise is AWGN

$\boxed{\frac{SNR_{out}}{SNR_{in}} = \frac{S^2(t)}{P_N B_o} \cdot \frac{P_N B_i}{S^2(t)} = \frac{B_i}{B_o}}$

From the BPF: $10 \log \frac{2\cdot 10^3}{10^6} = -33\,dB$

From the LPF: $10 \log \frac{10^1}{\frac{2\cdot 10^3}{2}} = -20\,dB$

Assuming SNR 1dB

(14.4)(a) We have $x_n = x_{n-1} + x_{n-4} \pmod 2$

Thus:

x_{n-4}	x_{n-3}	x_{n-2}	x_{n-1}	x_n
1	1	1	1	0
1	1	1	0	1
1	1	0	1	0
1	0	1	0	1
0	1	0	1	1
1	0	1	1	0
0	1	1	0	0
1	1	0	0	1
1	0	0	1	0
0	0	1	0	0

etc

(b) Age of universe $\simeq$ 10 billion years $= 10^{10}$ years $\simeq 10^{17}$ seconds

Assuming one cycle per nanosecond, we have

chip rate $(2^n - 1) \times 10^{-9}\,s = 10^{17}\,s \Rightarrow n \simeq 86$

(c) $CG = 10 \log_{10} \frac{SNR_{LFSR}}{SNR_{none}} = 10 \log_{10} \frac{2^{86}}{1} \simeq 260\,dB$

(14.5) $SQNR_{8bit} = 20 \log_{10} 2^8 \simeq 48\,dB$

$SQNR_{16bit} = 20 \log_{10} 2^{16} \simeq 96\,dB$

(14.6) Received: 00 10 01 11 00

The Trellis diagram for 5 states looks like this