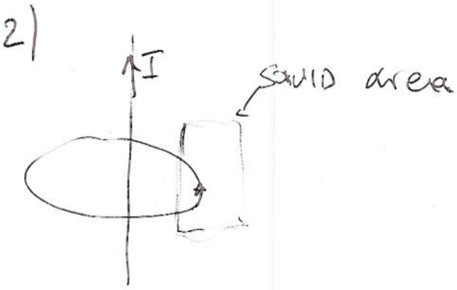
MAS. 862 P. I. T.
Chapter 15 Problem Set

(15.1) $E = 2E_F - 2E_c e^{-2/N_F V}$

we cannot differentiate around $V=0$!
Because $E(0) = 2E_F - 2E_c e^{-\frac{2}{0}}$ → no perturbation theory.

(15.2)



$$H = \frac{I}{2\pi r} \Rightarrow \frac{B}{\mu_0} = \frac{I}{2\pi r}$$

$$\rightarrow \frac{\phi}{A} = \frac{\mu_0 I}{2\pi r}$$

$$\Rightarrow r = \frac{\mu_0 I A}{2\pi \phi_0}$$

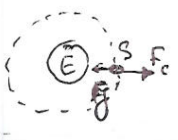
In our case $r = \frac{4\pi\times10^{-7}\,\frac{Tm}{A}\,1A \cdot 1cm^2}{2\cdot\pi\cdot 2.07\times10^{-7}\,T\,cm^2} \simeq 9.6\times10^3\,m$

(15.3) Seconds in a month: $2592000\,s$, $r_{earth} = 6.38\times10^6\,m$

So for Harrison's chronometer: $2592000\,s \cdot 2\pi \cdot 6.38\times10^6\,m \cdot \frac{10^{-5}}{84600\,s} \simeq 11\,km$

while for Cs atom clock $\left(\frac{10^{-12}}{1\,day}\right)$ it is $\simeq 1.2\,mm$

(15.5) (a) We want the centrifuge to be equal to the gravity force:



$$F_c = F_g \Rightarrow \frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{6.67\times10^{-11}\,m^3 kg^{-1} s^{-2} \times 5.98\times10^{24}\,kg}{\underbrace{2.02\times10^{7}\,m}_{\text{orbit}} + \underbrace{6.38\times10^{6}\,m}_{\text{earth radius}}}} \simeq 3.8\times10^3\,\frac{m}{s}$$

(15.5) (b) $v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\times\pi\times 2.66\times10^{7} m}{3.8\times10^{3} m\, s^{-1}} \simeq 4.31\times10^{4} s$

(c) $t_{earth} = 4.31\times10^{4} s$

$$t_{earth} - t_{sat} = t_{earth}\cdot\left(1-\sqrt{1-\frac{v^2}{c^2}}\right) = 4.31\times10^{4}\cdot\left(1-\sqrt{1-\underbrace{\frac{3.8\times10^{3}}{3\times10^{8}}}_{<1}}\right) \simeq 3.6\times10^{-6} s$$

<1 so sat clock runs slower

(d) Likewise:

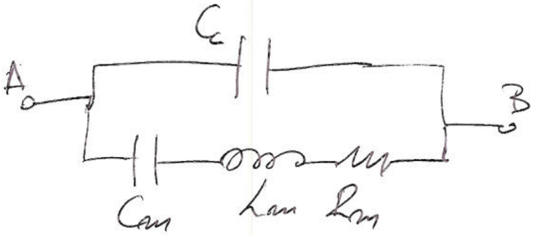
$$t_{earth} - t_{sat} = t_{earth}\left(1 - \frac{1-\frac{GM}{rc^2}}{1-\frac{GM}{rc^2}}\right)$$

$$= 4.31\times10^{4} s\cdot\left(1 - \frac{1-\frac{6.67\times10^{-11}\times 5.98\times10^{24}}{2.66\times10^{7}\times(3\times10^{8})^2}}{1-\frac{6.67\times10^{-11}\times 5.98\times10^{24}}{6.38\times10^{6}\times(3\times10^{8})^2}}\right)$$

$$\simeq -22.9\times10^{-6} s$$

$\rightsquigarrow$ sat clock runs faster

(15.4)



We have $Z_{AB} = Z_c \parallel Z_m = \frac{Z_c\cdot Z_m}{Z_c + Z_m} = \frac{\frac{1}{i\omega C_c}\cdot\left[\frac{1}{i\omega C_m} + i\omega L_m + R_m\right]}{\frac{1}{i\omega C_c} + \frac{1}{i\omega C_m} + i\omega L_m + R_m}$

In Laplace transform $Z(s) = \frac{\frac{1}{sC_c}\left[\frac{1}{sC_m} + sL_m + R_m\right]}{1/sC_c + 1/sC_m + sL_m + R_m} = \frac{[1 + s^2 C_m L_m + s C_m R_m]}{sC_m + sC_c + s^3 C_c C_m L_m + s^2 C_c C_m R_m}$

$$= \frac{s^2 C_m L_m + s C_m R_m + 1}{s^3 C_c C_m L_m + s^2 C_c C_m R_m + s(C_m + C_c)}$$

Analysis & plots continue in attached matlab file and publication

(/chap15prob4.m)