2.5 Problems

(2.1) (a) $1 \text{ yoctomole} = 10^{-24} \text{ mole}$

$1 \text{ mole} = 6.022 \times 10^{23} \text{ atoms}$ $\Rightarrow$ $1 \text{ ymole} = 0.6022 \text{ atom}$

(b) $1 \text{ nanocentury} = 10^{-9} \text{ centuries}$

$1 \text{ century} = 100 \text{ years}$

$1 \text{ year} = 365 \text{ days}$ $\Rightarrow$ $1 \text{ ncentury} = 3.1536 \text{ seconds} \simeq \pi \text{ seconds}$

$1 \text{ day} = 86400 \text{ seconds}$

(2.2) $1 \text{ Tbyte} = 10^{12} \text{ bytes}$

1 floppy disk (HD FDD) $= 1.44\,\text{MB} = 1.44 \times 10^{6}$ bytes $\Rightarrow$

$1 \text{ Tbyte} = 0.694 \times 10^{6}$ f.ds $= 694{,}000$ f.d.s $\Rightarrow$

1 f.d = 3.3 mm height

1 Tbyte of fds $= 2290.2$ m $\gg$ 1 km $\simeq$ Kingdom Tower largest skyscraper in Jeddah

(2.3) Universe $\simeq 10^{80}$ hydrogen atoms $\equiv 10^{80}$ bits

$$\text{largest number}_{(10)} = 1 + 2 + 4 + \dots + 2^{10^{78}} + 2^{10^{79}}$$
$$= 2^{0} + 2^{1} + 2^{2} + \dots + 2^{10^{78}} + 2^{10^{79}}$$
$$= 2^{10^{79}}\left[2^{10^{-79}} + 2^{10^{-78}} + \dots + 2^{10^{-1}} + 1\right]$$
$$\simeq 2^{10^{79}}\left[\underbrace{1 + 1 + \dots + 1 + 1}\right]$$

OK...

$$\boxed{2^{10^{80}} - 1}$$

$(2^{10}) \simeq 10^{3}$ so $10^{3 \times 79}$

(2.4) $g \simeq 9.81\ m/s^2$

$$g' = \frac{G \cdot 1kg}{1m^2} = \frac{6.673 \times 10^{-11} m^3 kg^{-1} s^{-2} \cdot 1kg}{1\ m^2} = 6.673 \times 10^{-11} m/s^2$$

$$\frac{g}{g'} = 20 \log_{10}\left(\frac{9.81}{6.673 \times 10^{-11}}\right) = 223 dB$$

(2.5) Let's find first how many moles is 1 ton of TNT

molar mass of TNT : $227.13\ g \cdot mol^{-1}$

$$\#mols = \frac{1.000.000}{227.13} = 4402.76 \text{ moles.}$$

Bond energies

N–O : 222 kJ/mol , N=O : 607 kJ/mol

in TNT we have

$3\times$ O_2N-C or 

result
$1.5 N \equiv N$
$= 946$ kJ

$$Energy_{BEF} = 4402.76 \times 3 [222 kJ + 607 kJ]$$
$$\approx 11 \times 10^6 kJ = 11 GJ$$

$$Energy_{AFT} = 4402.76 \times 1.5 \times 946 kJ = 6 GJ$$

$\Rightarrow$

Thermal energy = 11 GJ − 6 GJ = 5 GJ

Actual Number = 4.184 GJ (1 Ton)

calculation error: $\frac{5 - 4.184}{4.184} \approx 19\%$

(b) 10.000 T of TNT = 5×10^{15} J $\xrightarrow{/1.6 \times 10^{-19}}$ $\boxed{3.125 \times 10^{34} \text{ eV}}$

In uranium energy is stored in nucleus weak forces

(c) $E = m \cdot c^2 \Rightarrow m = \frac{E}{c^2} = \frac{41.84 \times 10^{15} \text{ J}}{9 \times 10^{16} \text{ m}^2/\text{s}^2}$ = ~~13.95 × 10^…~~

the reverse

$= 4.65 \times 10^{-1}$ kg $= 4.65 \times 10^2$ g

$= 465$ g of uranium.

→ 14.53×10^{28} atoms U

215 MeV

235 g/mole

569 g

kind of

DO IT with approximate bond energies

(2.6) (a) de Broglie wavelength of baseball

$\lambda = \frac{h}{p}$ $\qquad p_{\text{thrown baseball}} = 0.230 \text{ kg} \times 41 \text{ m/s} = 9.43 \text{ kg} \cdot \text{m/s}$

so $\lambda \simeq \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{9.43 \text{ kg m/s}} = 0.7 \times 10^{-34} \frac{\text{kg m}^2}{\text{s}^2} \cdot \frac{\text{s}}{\text{kg m/s}}$

$\Rightarrow \lambda \simeq 0.7 \times 10^{-34} \text{ m} = 0.7 \times 10^{-24}$ Å

(b) de Broglie wavelength of a molecule of nitrogen gas @ room T, P

we must find p (momentum)

$[PV = nRT]$ from equipartition Theorem:

3 degrees of freedom → $\frac{3}{2}kT$ thermal energy $\equiv \frac{1}{2} m v_{ave}^2$

$\Rightarrow p = m \cdot v_{ave} = \sqrt{3mkT}$

So $\lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{3 \cdot 14 \text{ g} \times (6.022 \times 10^{23})^{-1} \times k \times 300}} = 0.39$ Å

$\boxed{\text{atomic mass} \equiv \text{g/mole}}$

(c) Typical distance between molecules in above gas

\# of molecules in $V \to N_A$, where $V = 22.4$ liter in $\boxed{STP}$

$$\frac{V}{N_A} = \text{avery volume of single molecule}$$

lattice cube

$$d_1 d_2 d_3 = \frac{V}{N_A}$$

$$\Rightarrow \langle d \rangle^3 = \frac{V}{N_A} \rightarrow \langle d \rangle = \left(\frac{V}{N_A}\right)^{1/3} = \left(\frac{22.4\ ltr \times 10^{-3}\ m^3\ ltr^{-1}}{6.022 \times 10^{23}}\right)^{1/3}$$

$$= 33\ \text{Å}$$

OR sphere approx: $\frac{4}{3}\pi \langle r \rangle^3 = \frac{V}{N_A}$ $\quad N_A \cdot \frac{4}{3}\pi \langle r \rangle^3$

$$\Rightarrow \langle r \rangle = \left(\frac{3V}{4\pi N_A}\right)^{1/3} \simeq \overset{2\cdot}{1.61} \times 33\ \text{Å}$$

(d) $\lambda = \frac{h}{\sqrt{3mkT}} \leftarrow p$, p

I want $\lambda \simeq 33\ \text{Å}$ $\rightarrow \sqrt{3mkT} = \frac{33\ \text{Å}\ m}{6.626 \times 10^{-24}\ \text{Å}\ J \cdot s}$

$$\rightarrow 3mkT = 1/5 \times 10^{-48} \frac{J^2 s^2}{m^2}$$

$$\Rightarrow T = \frac{1/5 \times 10^{-48}\ J^2 s^2/m^2}{3 \cdot 0.014\,kg \cdot 6.022 \times 10^{-23} \cdot 1.38 \times 10^{-23}\ J/K}$$

$$\Rightarrow T = 0.57 \times 10^{-2} \frac{J^{2} \cdot s^{2} \cdot K}{m^2 \cdot kg^2 \cdot J}$$

$$\rightarrow T = 5.7\ mK$$

wrong calculations but correct approach

(2.7) $U_M = -\dfrac{GMm}{r}$

(a)

$C = \int u\,dr$

$F_M = -\dfrac{GMm}{r^2} \Rightarrow U_M = \int_0^r F_M\,dr = +\dfrac{GMm}{r}$

I want kinetic energy as much as the potential.

so $\dfrac{GMm}{r} = \frac{1}{2} m v^2 \rightarrow \boxed{v_{esc} = \sqrt{\dfrac{2GM}{r}}}$

(b) $c^2 = \dfrac{2GM}{r} \Rightarrow r_S = \dfrac{2GM}{c^2}$

Schwartschild radius horizon

e.g. earth mass $\simeq 6 \times 10^{24}$ kg

$r_{earth} = \dfrac{2 \cdot 6.67 \times 10^{-11} \times 6 \times 10^{24}}{9 \times 10^{16}}$ m

$\simeq 8.9 \times 10^{-3}$ m

(c) $\left.\begin{array}{l} E = M \cdot c^2 \\ E = \dfrac{h \cdot c}{\lambda} \end{array}\right\} \Rightarrow M c^2 = \dfrac{hc}{\lambda} \rightarrow \lambda = \dfrac{h}{M \cdot c}$

e.g. earth M $\rightarrow \lambda \simeq \dfrac{6.626 \times 10^{-34}}{6 \times 10^{24} \times 3 \cdot 10^8}$

$\rightarrow \lambda = 0.37 \times 10^{-66}$ m

Compton wavelength $\leftarrow$ OMG too small

(d) $\lambda = r_S \Rightarrow \dfrac{h}{M \cdot \cancel{c}} = \dfrac{2GM}{c^{\cancel{2}}} \rightarrow M^2 = \dfrac{c \cdot h}{2G} \Rightarrow M = \sqrt{\dfrac{c \cdot h}{2 \cdot G}} \simeq 2.176 \times 10^{-8}$ kg

PLANK MASS

maximum allowed mass for point masses

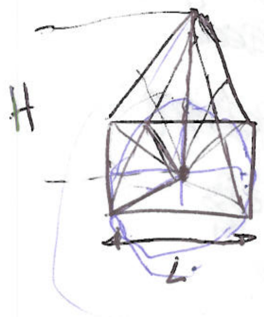
(e) $r_S \simeq \dfrac{2 \cdot 6.67 \times 10^{-11} \times 2.176 \times 10^{-8}}{9 \times 10^{16}} \simeq 1.05 \times 10^{-35}$ plank size

↳ factor of distance

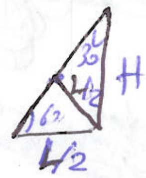
(f) Plank energy : $3.07 \times 10^{28}\, eV$ $\quad E = \frac{1}{2} m v^2 \quad T = \frac{\lambda}{v}$

(g) Plank time : $1.35 \times 10^{-43}\, s$ $\quad v_{esc} = \sqrt{\frac{2GM}{r_s}}$

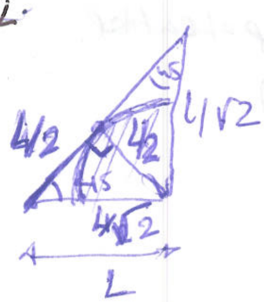
(2.8)



(a)



~~$\tan 60 = \frac{H}{L/2}$~~

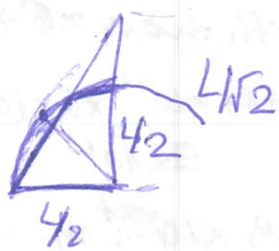


$\left(\frac{L}{2}\right)^2 + x^2 = \left(\frac{L}{\sqrt{2}}\right)^2$

$L/2$

$\tan 45 = \frac{H \cdot \sqrt{2}}{L}$ $\quad \rightarrow 1 = \frac{H}{L}\sqrt{2}$

$\rightarrow \boxed{H = \frac{L}{\sqrt{2}}}$



SMART

$\boxed{\pi \cdot r^2}$

$\pi \ell^2$

$\ell^2 =$

$y = \left(\frac{L^2}{4} - x^2\right.$

$x^2 =$