MAS.862 P.I.T.

Chapter 6 Problem Set

(6.1) $\vec{A}\times(\vec{B}\times\vec{C}) = \epsilon_{ijk} A_j (\epsilon_{klm} B_l C_m)$

$$= \epsilon_{ijk}\epsilon_{klm} A_j B_l C_m$$

$$\overset{(8.7)}{=} (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) A_j B_l C_m$$

$$= \delta_{il}\delta_{jm} A_j B_l C_m - \delta_{im}\delta_{jl} A_j B_l C_m$$

$$= A_j B_i C_j - A_j B_j C_i$$

$$= B_i (A_j C_j) - C_i (A_j B_j) \quad \text{(assuming } A_k B_k C_k \text{ commute)}$$

$$= B_i (\vec{A}\cdot\vec{C})_i - C_i (\vec{A}\vec{B})_i$$

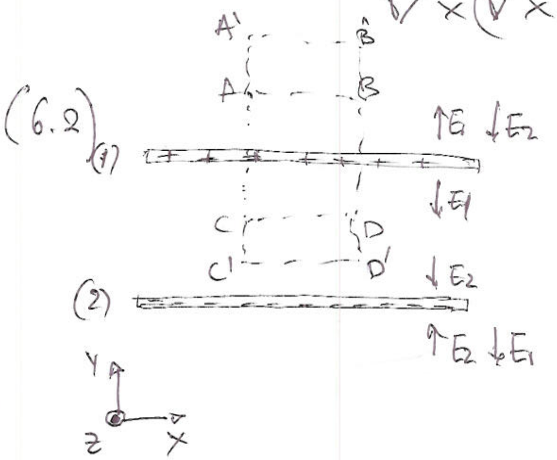
$$= \vec{B}\cdot(\vec{A}\cdot\vec{C}) - \vec{C}\cdot(\vec{A}\cdot\vec{B})$$

$\square$

Applying for $\vec{A}=\vec{B}=\vec{\nabla}$ we have

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{C}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{C}) - \vec{C}\,\underbrace{\vec{\nabla}^2}_{\vec{\nabla}^2 C}$$

(6.2)



By Gauss law we have the following observations:

- $\int_{AA'B'B} \vec{D}\cdot d\vec{A} = 0 \Rightarrow \vec{D}$ stable outside plates
- $\int_{CDD'D} \vec{D}\, d\vec{A} = 0 \Rightarrow \vec{D}$ stable inside plates
- For plate 1:

$$\int_{ABDC} \vec{D}_1 d\vec{A} = Q \Rightarrow \varepsilon \vec{E}_1 A = Q \Rightarrow$$ only $\hat{y}$ component by symmetry

$$\Rightarrow \vec{E}_1 = \begin{cases} \frac{Q}{\varepsilon A}\hat{y}, & Y \in \text{above plate 1} \\ \frac{Q}{\varepsilon A}(-\hat{y}), & Y \in \text{below plate 1} \end{cases}$$

- likewise for $\vec{E}_2$

Thus outside the plates $\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{Q}{\varepsilon A} - \frac{Q}{\varepsilon A}\right)\hat{y} = 0$

and inside the plates $\boxed{\vec{E} = \frac{2Q}{\varepsilon A}(-\hat{y})}$

Subsequently $V = -\int \vec{E}\,dl = \frac{2Q}{\varepsilon A}d$

and $C = \frac{Q}{V} = \frac{\varepsilon A}{2d}$

(b) By 2nd Maxwell's equation total current flowing through: $CDD'C'$

$$\int\left(\vec{J}_D + \frac{\partial\vec{D}}{\partial t}\right)d\vec{A} \overset{\vec{J}_D=0}{=} \int_{CDD'C'} \frac{\partial\vec{D}}{\partial t}\,d\vec{A}$$

$$\overset{\vec{B}\parallel CC' = B\parallel DD'}{=} \int_{CD'} \varepsilon\frac{\partial\vec{E}}{\partial t}\,d\vec{A}$$

$$= \varepsilon\,\frac{1}{d}\,\frac{dV}{dt}A$$

$$= C\frac{dV}{dt} = \frac{dQ}{dt} = I_D$$

(c) $U = \frac{1}{2}\int_V \vec{D}\cdot\vec{E}\,dV$

$= \frac{1}{2}\int_V \varepsilon E^2\,dV = \frac{1}{2}\varepsilon\frac{Q^2}{\varepsilon^2 A^2}dA = \frac{1}{2}CV^2$

(d) $P = 10\,V \cdot 10\,A = 100\,W = 100\,J/sec$

From above we have $100\,J/sec \cdot 3600\,sec = \frac{1}{2}C(10\,V)^2$

$$\Rightarrow C = 7200\,F$$

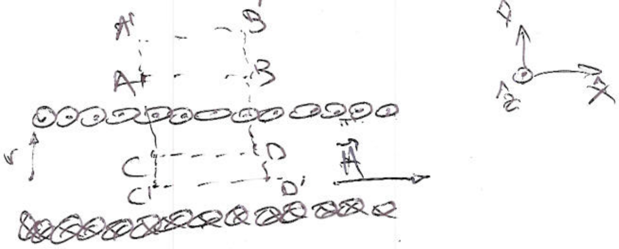
From (a): $7200\,F = \frac{\varepsilon A}{2d} = \frac{(8.854\times10^{-12}\,F.m^{-1})A}{2\cdot10^{-6}\,m} \rightarrow \boxed{A \simeq 16\times10^{8}\,m^2}$

If stacked on $(0.1\,m)^2$ plates: $\frac{16\times10^{8}\,m^2}{0.01\,m^2}\times10^{-6}\,m = 16\times10^{4}\,m$ height

!!!

a lot.

(6.3)



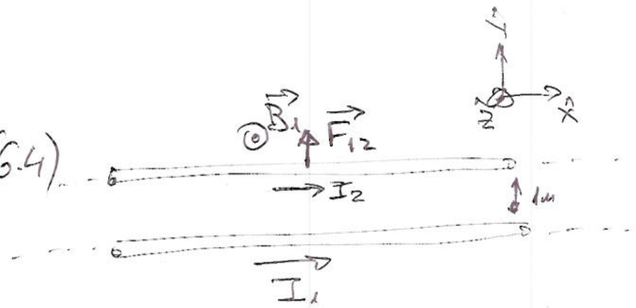
(a) With the same reasoning as (6.2) and Stokes law we have $\vec{H}=0$, stable outside the solenoid

and $\oint \vec{H}\, d\vec{l} = \int_S \vec{J}\, d\vec{A} \Rightarrow Hl = nlI \xRightarrow{\text{Symmetry}} \vec{H} = nI\hat{x}$ inside the solenoid.

(b) $U_m = 1/2 \int_V \vec{B}\cdot\vec{H}\, dV = 1/2 \int_V \mu H^2 dV = 1/2\, \mu n I \pi r^2 l$

(c) If the energy of the field is U_m then $F_m = \frac{\partial U_m}{\partial r} = \mu n I \pi r l = \pi r l B = \dots$

(6.4)



(Assuming infinite length)

By (6.86) we know that wire 1 generates a magnetic field

$$\vec{B}_1 = \frac{\mu I_1}{2\pi r}\hat{r} = \frac{\mu I_1}{2\pi r}\hat{z}$$

Then by the Laplace formula: $d\vec{F}_{12} = \vec{B}_1 \times I_2 d\vec{l} = \mu \frac{I_1 I_2}{2\pi r} dl$

So $\vec{F}_{12} = \frac{\mu I^2}{2\pi r}$ per meter

(6.5) To estimate the power transfered by sun's electromagnetic readiation I will average the value of the Poynting vector over a period.

$\langle|\vec{P}|\rangle = \langle|\vec{E}\times\vec{H}|\rangle = \langle|\sqrt{\frac{\epsilon_0}{\mu_0}}E^2|\rangle = \langle|\sqrt{\frac{\epsilon_0}{\mu_0}}E_0^2 e^{2i(\vec{k}x-\omega t)}|\rangle = \sqrt{\frac{\epsilon_0}{\mu_0}}E_0^2 \langle|e^{2i(\vec{k}x-\omega t)}|\rangle$

But for $t=T$ $|e^{i2\vec{k}x}| = 1$ and $|e^{-2i\omega t}| = \langle|\cos 2\omega T - i\sin 2\omega T|\rangle = \langle|\cos 2\pi - i\sin 2\pi|\rangle$

$= 1/2$

thus $\langle|\vec{P}|\rangle = 1/2\sqrt{\frac{\epsilon_0}{\mu_0}}E_0^2$

So we have $10^3\ W\ m^{-2} = \int_{1m^2} \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2\, dA$

or $E_0^2 = \dfrac{2\times 10^3\ W}{\sqrt{\frac{\varepsilon_0}{\mu_0}} \times 1\ m^2} = 377 \times 2\ 10^3\ V^2$ (1)

or $E_0 \simeq 868\ V$

(b) Using (1) $E_0^{(1)} = \sqrt{\dfrac{2\times 1\ W}{\sqrt{\frac{\varepsilon_0}{\mu_0}}\,(10^{-3})^2\, m^2}} = \sqrt{377 \times 2 \times 10^6\ V^2} = 27459\ V = 27\ kV$

$E_0^{(2)} = \sqrt{\dfrac{2\times 1\ W}{\sqrt{\frac{\varepsilon_0}{\mu_0}}\,(10^{-6})^2\, m^2}} = \sqrt{377 \times 2 \times 10^{12}\ V^2} \simeq 27\ MV$