

MAS. 862 P. I. T.

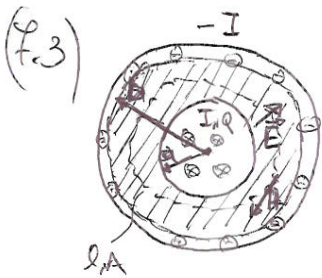
Chapter 7 Problem Set

(7.1) In order to be able to cancel the noise in a differential mode, the two wires have to be exposed equally to the noise source. In long cable pairs (like the ones in telecom or the grid) each cable acts as an inductor and interferes with the nearby cables, thus introducing a non differentiable noise. By twisting the pairs, we mitigate this effect by equalizing the magnetic flux exchange between the two wires.

A grounded shield minimizes capacitively coupled noise from external nearby sources. (electrostatic screening: Faraday cage)

(7.2) By direct application of 7.33

$$\delta = \frac{1}{\sqrt{\omega \mu \sigma}} = \frac{1}{\sqrt{3.14159 \times 10^4 \times 4 \times 3.14159 \times 10^{-7} \times 4}} = 3.97 \text{ m}$$



From Gauss Law on A we have $\vec{E} = \frac{Q}{2\pi \epsilon r} \hat{r}$

From Stokes Law on l we have $\vec{H} = \frac{I}{2\pi r} \hat{\phi}$

$$\text{So: } \vec{P} = \vec{E} \times \vec{H} = \frac{Q}{2\pi \epsilon r} \hat{r} \times \frac{I}{2\pi r} \hat{\phi} = \frac{Q I}{4\pi^2 \epsilon r^2} \hat{z}$$

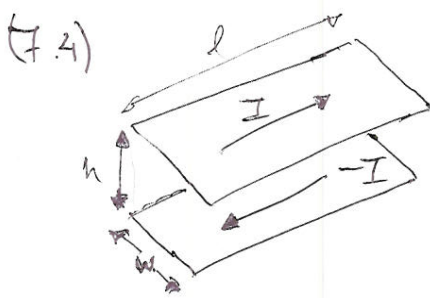
Integrating over r and ϕ : $P = \int_0^{2\pi} \int_a^B \frac{Q I}{4\pi^2 \epsilon r^2} r dr d\phi$ (a : inner radius)
(B : outer radius)

$$= \frac{Q I}{2\pi \epsilon} \int_a^B \frac{1}{r} dr$$

$$= \frac{Q I}{2\pi \epsilon} \ln \frac{B}{a}$$

$$= I \cdot \left(\frac{Q}{2\pi \epsilon} \ln \frac{B}{a} \right)$$

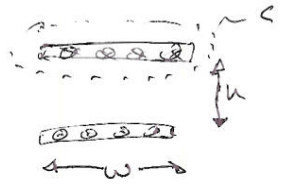
$\underbrace{\hspace{2cm}}_V$



To find characteristic impedance, I need to find the capacitance and inductance of the strip.

Assuming a parallel plate model:

we have $C = \frac{\epsilon w}{h} \text{ F/m}$ (from 5.1)



Then by Stokes law around one plate (curve C) we

have $\int_C \vec{H} \cdot d\vec{l} = I \Rightarrow Hw = I \Rightarrow \boxed{H = \frac{I}{w}}$

So we have a magnetic flux $\Phi = \int \vec{B} \cdot d\vec{A} = \int \mu \frac{I}{w} dA = \mu \frac{I}{w} h l$

and thus $L = \frac{\Phi}{I l} = \mu \frac{h}{w} \text{ H/m}$

Now we have $Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu h h}{w \epsilon w}} = \frac{h}{w} \sqrt{\frac{\mu}{\epsilon}}$

and $v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{\mu h}{w} \frac{\epsilon w}{h}}} = \frac{1}{\sqrt{\mu \epsilon}}$ which is the correct speed!

(7.5)
 $\epsilon_r = 2.26$
 $a = 0.406 \text{ mm}$
 $B = 1.48 \text{ mm}$

(a) $Z = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu_0 \ln \frac{B}{a}}{2\pi} \frac{1}{2\pi \epsilon_r \epsilon_0} \frac{\ln \frac{B}{a}}{a}}$
 $= \frac{1}{2\pi} \ln \frac{B}{a} \frac{1}{\sqrt{\epsilon_r}} \sqrt{\frac{\mu_0}{\epsilon_0}}$
 $= \frac{1}{2 \cdot \pi} \ln \left(\frac{1.48}{0.406} \right) \frac{1}{\sqrt{2.26}} \cdot 377$
 $\approx 50 \Omega$

(b) $v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{\mu_0 \ln \frac{B}{a}}{2\pi} \frac{2\pi \epsilon_r \epsilon_0}{\ln \frac{B}{a}}}} = \frac{1}{\sqrt{\mu_0 \epsilon_r \epsilon_0}} = \frac{1}{\sqrt{\epsilon_r}} \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{c_0}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8 \text{ m/s}}{\sqrt{2.26}} \approx 2 \times 10^8 \text{ m/s}$

(c) $l \leq v \cdot t \leq 2 \times 10^8 \text{ m/s} \cdot 10^{-9} \text{ sec} = 0.2 \text{ m}$

(d) We want $Z = 50 \Omega$ with $\beta = 30 \text{ mils} = 0.762 \text{ mm}$

$$\text{from (a): } 50 \Omega = \frac{1}{2\pi} \ln\left(\frac{0.762}{a}\right) \frac{1}{\sqrt{2.26}} 377$$

$$\rightarrow \ln\left(\frac{0.762}{a}\right) = 1.2527$$

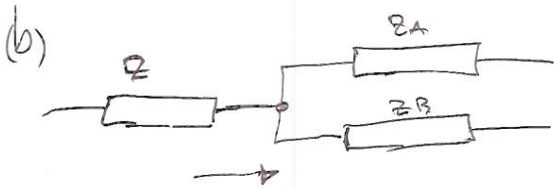
$$\rightarrow a = \frac{0.762}{e^{1.2527}} = 0.218 \text{ mm} \approx 8.6 \text{ mils}$$

(e) we want $\lambda = \beta \cdot 2 = 2.96 \text{ cm} = 2.96 \times 10^{-3} \text{ m}$

$$\text{then } f = \frac{v}{\lambda} = \frac{2 \times 10^8 \text{ m/s}}{2.96 \times 10^{-3} \text{ m}} \approx 67.5 \text{ GHz}$$

(7.6) From (7.5) we have the transmission velocity $v \approx 2 \times 10^8 \text{ m/s}$

$$(a) \lambda_{\text{bit}} = v_{\text{bit}} \cdot t_{\text{bit}} = 2 \times 10^8 \text{ m/s} \cdot \frac{1}{10^7} \text{ s/bit} = 20 \text{ cm/bit}$$



looking from the single line we see load impedance:

$$\frac{1}{Z_L} = \frac{1}{Z_A} + \frac{1}{Z_B} \Rightarrow Z_L = \frac{Z_A Z_B}{Z_A + Z_B} = \frac{50 \cdot 50}{2 \cdot 50} = 25 \Omega$$

$$\text{thus } |R| = \left| \frac{Z_L - Z}{Z_L + Z} \right| = \left| \frac{25 - 50}{25 + 50} \right| = 1/3$$