

2.1 (a) No. of atoms in one mole = 6.022×10^{23} (1)

$$1 \text{ yoctomole} = 10^{-24} \times 6.022 \times 10^{23}$$

$$\text{No. of atoms in 1 yoctomole} = 0$$

(b) 1 century = 100 years = $365 \times 24 \times 60 \times 60 \times 100$
 $= 31.536 \times 10^8$ seconds

$$1 \text{ nanocentury} = 31.536 \times 10^8 \times 10^{-9} \text{ seconds}$$

$$= \cancel{0.031536} \ 3.1536 \text{ seconds}$$

It is close to the value of π $\left(\frac{\text{Circumference of circle}}{\text{Diameter of circle}} \right)$

2.2. Average capacity of one CD = 700 MiB

$$= 700 \text{ Mibi byte}$$

$$= 700 \times 1048576 \text{ bytes}$$

$$= 7.34 \times 10^8 \text{ bytes}$$

$$\text{No. of CDs equivalent to 1 petabyte} = \frac{10^{15}}{7.34 \times 10^8}$$

$$= 1.36 \times 10^6$$

$$\text{Thickness of each CD} = 1.2 \text{ mm}$$

$$\text{Height of a stack of } 1.36 \times 10^6 \text{ CDs} = 1.2 \times 10^{-3} \text{ m} \times 1.36 \times 10^6$$

$$= 1.632 \times 10^3 \text{ m}$$

$$= \underline{\underline{1632 \text{ m}}}$$

2.3 Approximate no. of atoms in universe = 10^{80} (2)

If each atom represents a bit,

10^{80} bits represent numbers upto $(2)^{10^{80}}$

$$x = (2)^{10^{80}}$$

$$\log_{10} x = \log_{10} (2)^{10^{80}}$$

$$= 10^{80} \log_{10} (2)$$

$$= 10^{80} \times 0.301$$

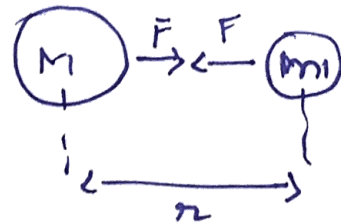
$$\log_{10} x = 3.01 \times 10^{79}$$

$$x = 10^{3.01 \times 10^{79}}$$

$$\underline{\underline{x = 10^{(3 \times 10^{79})}}}$$

2.4 Gravitational force between bodies of mass

M and m' = $G \frac{M \times m'}{r^2}$ where r is the distance between masses.



Gravitational force on a mass m' at the surface of earth = $\frac{G \times 5.98 \times 10^{24} \text{ kg} \times m'}{(6.378 \times 10^6)^2}$

Gravitational force on a mass m' produced by 1kg at distance 1m = $\frac{G \times 1 \text{ kg} \times m'}{1}$

2.4) $F \propto$ Acceleration

(3)

$$\text{Ratio of acceleration} = \frac{6 \times 5.98 \times 10^4 \times \text{m}'}{(6.378 \times 10^6)^2} \bigg/ \frac{6 \times 1 \text{ kg} \times \text{m}'}{1}$$

$$= \frac{5.98 \times 10^4}{(6.378 \times 10^6)^2} = \frac{5.98 \times 10^4}{40.68 \times 10^{12}}$$

$$= 0.123 \times 10^{-8} = 1.23 \times 10^{-9}$$

$$\text{Ratio in decibels} = 20 \log_{10}(1.23 \times 10^{-9})$$

$$= 20 [\log_{10} 1.23 + \log_{10} 10^{-9}]$$

$$= 20 [0.0899 - 9]$$

$$\approx 20 \times -9$$

Ratio

$$\approx \underline{\underline{-180 \text{ dB}}}$$

2.7 (a) To climb out of potential energy $-\frac{GMm}{r}$

kinetic energy $>$ potential energy.

$$\frac{1}{2}mv^2 > \left| -\frac{GMm}{r} \right|, \text{ where } v = \text{escape velocity.}$$

$$\frac{1}{2}v^2 > \left| -\frac{GM}{r} \right|$$

$$v^2 > \frac{2GM}{r}, \quad v > \sqrt{\frac{2GM}{r}}$$

(b) Nothing can be faster than light,
hence $v \leq c$, c is speed of light.

$$\sqrt{\frac{2GM}{r}} < v \leq c \Rightarrow \sqrt{\frac{2GM}{r}} \leq c.$$

$$\frac{2GM}{r} \leq c^2, \quad r \geq \frac{2GM}{c^2}.$$

For radii $r < \frac{2GM}{c^2}$ nothing can escape the mass.

(c) Rest energy of mass $M = Mc^2$.

Energy of photon of wavelength $\lambda = \frac{hc}{\lambda}$

$$\frac{hc}{\lambda} = Mc^2, \quad \boxed{\lambda = \frac{h}{Mc}}$$

2.7 (d) Given $\lambda = \frac{2GM}{c^2}$ from 2.7 b.

$$\frac{2GM}{c^2} = \frac{h}{Mc} \quad \text{from 2.7. c}$$

$$M^2 = \frac{hc}{2G}, \quad M = \sqrt{\frac{hc}{2G}}$$

(e) $\pi = \frac{2GM}{c^2} = \frac{2G}{c^2} \sqrt{\frac{hc}{2G}}$

$$= \frac{1}{c} \sqrt{\frac{hc}{2G}}$$

(f) Energy = $\left| \frac{GMm}{\pi} \right| = \left[\frac{GM \cdot \sqrt{\frac{hc}{2G}}}{\frac{2G}{c^2} \sqrt{\frac{hc}{2G}}} \right]$

$$= \frac{1}{2} M c^2 = \frac{1}{2} \sqrt{\frac{hc}{2G}} \cdot c^2$$

(g) $E = h\nu$, $\nu = E/h$, period = $\frac{h}{E}$