6.1) $\vec{A}\times(\vec{B}\times\vec{C})$

$$\vec{B}\times\vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = (B_2C_3 - B_3C_2)\hat{x} - [B_1C_3 - B_3C_1]\hat{y} + (B_1C_2 - B_2C_1)\hat{z}$$

$$\vec{A}\times(\vec{B}\times\vec{C}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_1 & A_2 & A_3 \\ (B_2C_3 - B_3C_2) & (B_3C_1 - B_1C_3) & (B_1C_2 - B_2C_1) \end{vmatrix}$$

$$= (A_2B_1C_2 - A_2B_2C_1 - A_3B_3C_1 + A_3B_1C_3)\hat{x}$$
$$- (A_1B_1C_2 - A_1B_2C_1 - A_3B_2C_3 + A_3B_3C_2)\hat{y}$$
$$+ (A_1B_3C_1 - A_1B_1C_3 - A_2B_2C_3 + A_2B_3C_2)\hat{z}$$

$$= \begin{pmatrix} \underline{A_1C_1B_1} + A_2B_1C_2 + A_3B_1C_3 \\ \underline{-A_1C_1B_1} - A_2B_2C_1 - A_3B_3C_1 \end{pmatrix}\hat{x}$$

$$- \begin{pmatrix} A_1B_1C_2 + \underline{A_2B_2C_2} + A_3B_3C_2 \\ -A_1B_2C_1 - \underline{A_2B_2C_2} - A_3B_2C_3 \end{pmatrix}\hat{y}$$

$$+ \begin{pmatrix} A_1C_1B_3 + A_2C_2B_3 + \underline{A_3C_3B_3} \\ -A_1B_1C_3 - A_2B_2C_3 - \underline{A_3C_3B_3} \end{pmatrix}\hat{z}$$

$= B_1 (A_1C_1 + A_2C_2 + A_3C_3)\hat{x} - C_1(A_1B_1 + A_2B_2 + A_3B_3)\hat{x}$
$- C_2(A_1B_1 + A_2B_2 + A_3B_3)\hat{y} + B_2(A_1C_1 + A_2C_2 + A_3C_3)\hat{y}$
$+ B_3(A_1C_1 + A_2C_2 + A_3C_3)\hat{z} - C_3(A_1B_1 + A_2B_2 + A_3B_3)\hat{z}$

$= (B_1\hat{x} + B_2\hat{y} + B_3\hat{z})(A_1C_1 + A_2C_2 + A_3C_3)$
$- (C_1\hat{x} + C_2\hat{y} + C_3\hat{z})(A_1B_1 + A_2B_2 + A_3B_3)$

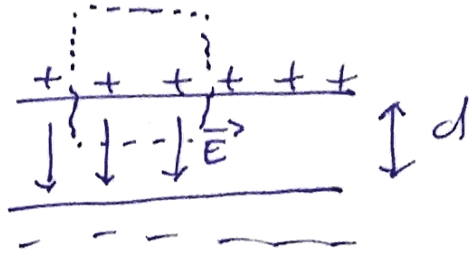
$= (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C} = \underline{\underline{\vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})}}$

Now $\nabla \times (\nabla \times \vec{E})$, ~~here~~ $\nabla \cancel{\times} \cancel{\vec{A}}$,

$\nabla \times (\nabla \times \vec{E}) \equiv (\vec{A} \times (\vec{B} \times \vec{C})) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

$= \underline{\underline{\nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E}}}$

6.2 (a) From Gauss's law



$$\int_v \nabla\cdot\vec{E}\, dv = \int_s \vec{E}\cdot d\vec{A}$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon}$$

$$\int \frac{\rho}{\epsilon}\, dv = \int_s \vec{E}\cdot d\vec{A}$$

$$\int_s \vec{E}\cdot d\vec{A} = \frac{Q}{\epsilon}$$

$$\frac{Q}{\epsilon} = EA$$

$$E = \frac{Q}{\epsilon A}$$

$$V = -\int \vec{E}\cdot d\vec{l}$$

$$V = -Ed. = \frac{Qd}{\epsilon A}$$

$$Q = \frac{\epsilon A V}{d}$$

$$C = \frac{Q}{V} = \frac{\epsilon A \cancel{v}}{d} \Big/ \cancel{v}$$

$$\boxed{C = \frac{\epsilon A}{d}}$$

6.2 ⟨b⟩ $\int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{A} = \int \epsilon \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$

$$= \epsilon \frac{\partial}{\partial t}\left(\frac{v}{d}\right) \cdot A$$

$$= \frac{\epsilon A}{d} \frac{\partial v}{\partial t}.$$

$$= C \frac{\partial v}{\partial t}$$

$= I =$ external current

6.2 ⟨c⟩ Energy density of capacitor $\approx \vec{D} \cdot \vec{E}$.

Stored energy $U = \frac{1}{2} \int_V \vec{D} \cdot \vec{E}\, dv$

$$= \frac{1}{2} \int_V \epsilon E^2\, dv$$

$$= \frac{1}{2} \epsilon \int_V \frac{Q^2}{\epsilon^2 A^2}\, dv$$

$$= \frac{1}{2} \cdot \frac{Q^2}{\epsilon A^2} \int_V dv = \frac{1}{2} \frac{Q^2}{\epsilon A^2} \cdot Ad$$

$$= \frac{1}{2} \frac{\epsilon A}{d} \cdot \frac{Q^2}{\epsilon^2 A^2} d^2 = \frac{1}{2} C (E^2 d^2)$$

$$= \frac{1}{2} C V^2$$

6.2) (d)

Energy $= \frac{1}{2} CV^2 = \frac{1}{2} C\, 10^2 = 50C$ J

Power delivered by battery $= 10V \times 10A = 100W$

Total energy $= 100W \times 60 \times 60$

$= 3.6 \times 10^5$ J

$50C = 3.6 \times 10^5$ J

$\Rightarrow C = \frac{3.6 \times 10^4}{5}$ F $= 7200$ F

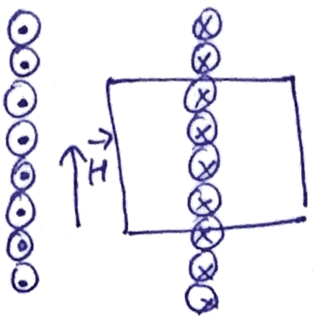
$C = \frac{\epsilon A}{d}$ $\Rightarrow A = \frac{Cd}{\epsilon} = \frac{7200 \times 10^{-6}}{8.854 \times 10^{-12} F/m}$

$= \underline{\underline{8 \times 10^8 \ m^2}}$

If one side is 10cm ~~other side =~~ $\frac{8 \times 10^8}{(0.1)^2} = 8 \times 10^{10}$
they are separated by 10^{-6} m

Hence height of stack $= \frac{8 \times 10^8}{(0.1)^2} \times 10^{-6} = 8 \times 10^4$ m

5.3(a)



From Stoke's law.

$$\int \vec{H}\cdot d\vec{l} = \int_S \vec{J}\cdot d\vec{A}$$

$\vec{H}$ is uniform inside solenoid.

$$Hl = \int_S \vec{J}\cdot dA$$

n wires per metre $\Rightarrow$ nl wires for overall length.

$\Rightarrow \int_S \vec{J}\cdot dA = Inl$, this gives $Hl = Inl$.

$$\boxed{H = nI \quad \text{or} \quad B = \mu H = \mu n I}$$

5.3)(b) Energy stored $U = \frac{1}{2}\int_V \vec{B}\cdot\vec{H}\, dv$

$$= \frac{1}{2}\int_V \mu\vec{H}\cdot\vec{H}\, dv = \frac{\mu}{2}\int_V H^2 dv = \frac{\mu}{2}\int n^2 I^2 dv.$$

$$= \frac{1}{2}\mu n^2 I^2 \pi r^2 l$$

(c) Force $= \dfrac{\partial U}{\partial r}$ = gradient $= \mu n^2 I^2 \pi 2 r l \cdot \frac{1}{2}$

$$= \mu n^2 I^2 \pi r l.$$

$$\text{Force} = (\pi r l)\ \mu n^2 I^2$$

$$= (\pi r l)\ \frac{\mu^2 n^2 I^2}{\mu} \quad , \text{ But } B = \mu n I.$$

$$= \pi r l\ B$$

$$= \frac{\pi \times 0.5 \times 2 \times 10^2}{4\pi \times 10^{-7}}\ N$$

$$= 0.25 \times 10^9\ N$$

$$= \underline{\underline{2.5 \times 10^9\ N}}$$

6.4) $F = I\,\vec{dl} \times \vec{B}$

$= I\,\vec{dl} \times \mu\vec{H}$

$= I\,dl\,\mu \dfrac{I}{2\pi r}$

$F = \dfrac{\mu I^2}{2\pi r}\,dl$

$F/dl = \dfrac{\mu I^2}{2\pi r} = \dfrac{4\pi \times 10^{-7}\ H/m \cdot (1A)^2}{2\pi \times 1}$

Force per meter $= \underline{\underline{2 \times 10^{-7}}}$ N