8.1) $$\vec{E} = \frac{1}{i\omega\mu_0\varepsilon_0}\nabla(\nabla\cdot\vec{A}) - i\omega A$$

For infinitesimal dipole

$$A_r = \mu_0 \frac{I_0 d\, e^{-kr}}{4\pi r}\cos\theta \qquad A_\theta = \frac{\mu_0 I_0 d e^{-ikr}\sin\theta}{4\pi r}$$

$$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta A_\theta)$$

$$= -\frac{e^{-kr}\mu_0 I_0 d}{4\pi}\left[\frac{1}{r^2} + \frac{ik}{r}\right]\cos\theta$$

$$\nabla(\nabla\cdot\vec{A})_\theta = \frac{1}{r}\frac{\partial}{\partial\theta}(\nabla\cdot\vec{A})$$

$$= \frac{\mu_0 I_0 d e^{-kr}\sin\theta}{4\pi}\left[\frac{1}{r^3} + \frac{ik}{r^2}\right]$$

$$\nabla(\nabla\cdot\vec{A})_r = \frac{\partial}{\partial r}(\nabla\cdot\vec{A})$$

$$= \frac{\mu_0 I_0 d e^{-kr}\cos\theta}{4\pi}\left[\frac{-k^2}{r} + \frac{2}{r^3} + \frac{2ik}{r^2}\right]\cos\theta$$

$$E_\theta = \frac{\mu_0 q_0 d\, e^{-ikr}}{(i\omega\mu_0\varepsilon_0)\, 4\pi} \left[\frac{1}{r^3} + \frac{ik}{r^2} \right] \sin\theta$$

$$+ \frac{i\omega\mu_0 q_0 d \sin\theta}{4\pi r} e^{-ikr}$$

$$= \frac{q_0 d e^{-ikr}}{4\pi} \sin\theta \left[\frac{i\omega\mu_0}{r} + \frac{1}{r^2}\sqrt{\frac{\mu_0}{\varepsilon_0}} + \frac{1}{i r^3 \omega\varepsilon_0} \right]$$

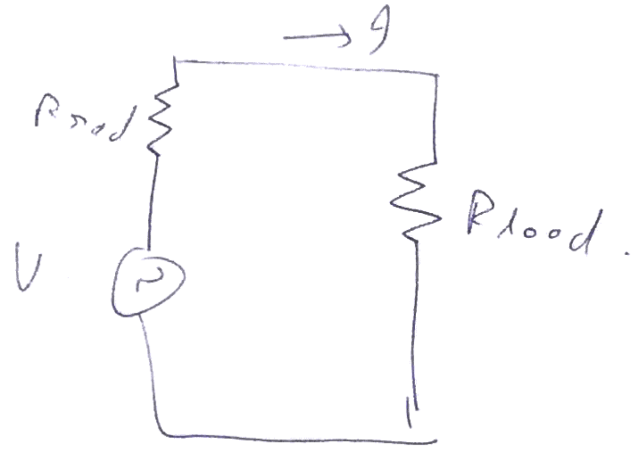
$$E_r = \frac{q_0 d e^{-ikr}}{4\pi} \left[\frac{2}{r^2}\sqrt{\frac{\mu_0}{\varepsilon_0}} + \frac{2}{i\varepsilon_0\omega r^3} \right] \cos\theta$$

8.2) we have $\vec{P} = \vec{E} \times \vec{H} = \vec{E} \times \left(\sqrt{\frac{\varepsilon_0}{\mu_0}}\, \hat{k} \times \hat{E} \right)$

$\langle P \rangle = \frac{1}{2} \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E^2$

$\langle P \rangle = \frac{1\,kW}{4\pi (1000)^2}$

8.3



$$P = V_{load} I_{load}$$

$$\Rightarrow I_{load} = \frac{V}{R_{rad} + R_{load}}, \quad V_{load} = \frac{R_{load} V}{R_{rad} + R_{load}}$$

$$P = \frac{V^2 R_{load}}{(R_{rad} + R_{load})}$$

Maximizing power with respect to Rload.

$$\frac{dP}{dR_{load}} = 0 = \frac{V^2}{(R_{rad} + R_{load})^2} - \frac{2 R_{load} V^2}{(R_{rad} + R_{load})^3}$$

$$\frac{2 R_{load}}{R_{rad} + R_{load}} = 1$$

$$\boxed{R_{lod} = R_{rad}}$$