8.1/ $\vec{E}$ of infinitesimal dipole?

From book, $\vec{E} = \frac{1}{i\omega\mu_0\varepsilon_0} \nabla(\nabla\cdot\vec{A}) - i\omega\vec{A}$

and $\vec{A}_\theta = \mu_0 \frac{I_0 d e^{-ikr}}{4\pi r} \sin\theta$

$\vec{A}_r = \mu_0 \frac{I_0 d e^{-ikr}}{\pi r} \cos\theta$

$\nabla\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta \sin\theta)$

from def of ∇ del in spherical

$= \mu_0 \frac{I_0 d}{\pi} e^{-ikr}\left[\frac{1}{r^2} + \frac{ik}{r}\right]\cos\theta$

We still need to calculate $\nabla(\nabla\cdot\vec{A})$

$\nabla_\theta(\nabla\cdot\vec{A}) = \frac{1}{r}\frac{\partial}{\partial\theta}(\cdot) = \frac{\mu_0 I_0 d}{4\pi} e^{-ikr}\left[\frac{1}{r^3} + \frac{ik}{r^2}\right]\sin\theta$

$\nabla_r(\nabla\cdot\vec{A}) = \frac{\partial}{\partial r}(\cdot) = \ldots\left[\frac{2ik}{r^2} - \frac{k^2}{r} + \frac{2}{r^3}\right]\cos\theta.$

Combining all of this,

$E_\theta = \ldots$

algebra...

$E_r = \ldots$

8.2/ From 8.31 in book $W = \int \vec{P} \cdot d\vec{A}$

$\Rightarrow W = \langle P \rangle -$

$\Rightarrow \langle P \rangle = \frac{W}{A} = \frac{W}{4\pi r^2}$

$\langle P \rangle = \frac{10^3}{4\pi (10^3)^2} = 7.96 \times 10^{-5} \, W/m^2$

$\vec{P} = \vec{E} \times \vec{H}$

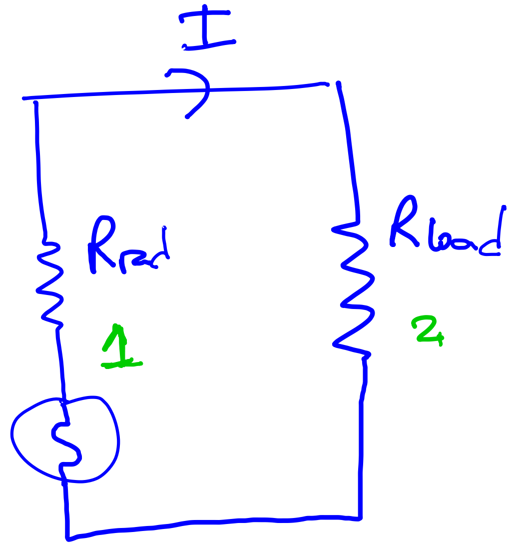
$= \vec{E} \times \underbrace{\sqrt{\frac{\varepsilon_0}{\mu_0}} \hat{k} \times \vec{E}}$ (eq 8.38, 6.102 in book)

$\langle P \rangle = \sqrt{\frac{\varepsilon_0}{\mu_0}} \underbrace{\langle E^2 \rangle}_{\frac{1}{2} E_{peak}}$

$E_{peak} = \left(2 \langle P \rangle \sqrt{\frac{\mu_0}{\varepsilon_0}} \right)^{1/2} = \ldots$

8.3/

$$I = \frac{V}{R_{rad} + R_{load}}$$



$$P_{load} = I^2 R_{load}$$

$$= V^2 \frac{R_{load}}{(R_{rad} + R_{load})^2}$$

P_{load} is max/min when $\frac{\partial P_{load}}{\partial R_{load}} = 0$ when $R_{rad} = R_{load}$

8.4/

$$G = \max_{\theta, \varphi} \frac{P(r=1, \theta, \varphi)}{W/4\pi}$$

From notes, $P = \underbrace{\frac{I^2 k^2 d^2}{32 \pi^2 r^2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \sin^2\theta}_{\text{max}} \, \hat{r}$

$$W = \frac{I_0^2 \pi}{3} \sqrt{\frac{\mu_0}{\varepsilon_0}} \left(\frac{d}{\lambda}\right)^2$$

$$G = \frac{\cdots}{\cdots} \times 4\pi$$

$$= 3/2$$