6.1/ $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

a) $[B\times C]_k = \varepsilon_{klm} B_l C_m$

$$[A\times(B\times C)]_i = \varepsilon_{ijk} A_j \left(\varepsilon_{klm} B_l C_m\right)_k$$

already for k

$$= \varepsilon_{ijk}\varepsilon_{klm} A_j B_l C_m$$

$$= \left[\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}\right] A_j B_l C_m$$

$$= \delta_{il}\underbrace{\delta_{jm} A_j B_l C_m}_{j=m} - \delta_{im}\underbrace{\delta_{jl} A_j B_l}_{j=l \text{ only}} C_m$$

everything else zero

$$= \underbrace{\delta_{il} A_j B_l}_{i=j \text{ only}} C_j - \delta_{im} A_j B_j C_m$$

$$= A_j B_i C_j - A_j B_j C_i$$

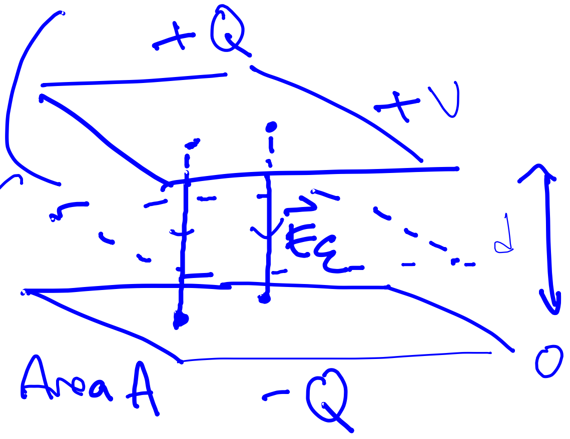
$$= B_i(A\cdot C) - C_i(A\cdot B) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$$

b) $\overset{A}{\nabla}\times(\overset{B}{\nabla}\times\overset{C}{\vec{E}}) = \underbrace{\overset{B}{\nabla}(\overset{A}{\nabla}\cdot\overset{C}{\vec{E}})}_{\text{same}} - \vec{E}\,(\nabla\cdot\nabla)$

$$\nabla\cdot\nabla = \partial_i\partial_i = \partial_i^2$$

6.2 a) Gauss Law: $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$ (diff form)

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$



$$\int \varepsilon \vec{E}\, dA = 0$$ (no charge in between plates.)

(field entering area = field leaving)

$$\int \varepsilon E\, d\vec{A} = Q$$

thru plate

$$\Rightarrow E = \frac{Q}{\varepsilon A}$$

$$V = -\int \vec{E} \cdot d\ell = Ed = \frac{Q}{\varepsilon A} d$$

$$C \underset{\text{defn}}{=} \frac{Q}{V} = \frac{\varepsilon A}{d}$$

b) $E = \frac{V}{d}$

$$\int \frac{\partial \vec{D}}{\partial t} d\vec{A} = \int \varepsilon \frac{\partial \vec{E}}{\partial t} dA$$

$$= \frac{\varepsilon}{d} \int \frac{\partial \vec{V}}{\partial t} dA = \frac{\varepsilon A}{d} \frac{dV}{dt}$$

$$= C \frac{d}{dt} = \frac{dQ}{dt} = I$$

but this is constant here??

c) $= \frac{1}{2}\left(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H}\right)$ $\frac{J}{m^3}$ energy density.

$$U = \frac{1}{2}\int_V \vec{E}\cdot\vec{D}\,dV = \frac{1}{2}\int \varepsilon E^2 dV$$

$V = Ad$
$dV = dA\cdot d$

$$= \frac{1}{2}\varepsilon d\int E^2 dA = \frac{\varepsilon}{2}\int\left(\frac{Q}{\varepsilon A}\right)^2 dA$$

$$= \frac{\varepsilon d}{2}\frac{Q^2}{\varepsilon^2 A^2}A = \frac{1}{2}d\frac{Q^2 d}{\varepsilon A}$$

$$= \frac{1}{2}d\underbrace{\frac{\varepsilon A}{d}}_{C}\frac{d}{\varepsilon A}\frac{Q^2}{\varepsilon A}$$

$$= \frac{1}{2}C\underbrace{\frac{d^2 Q^2}{\varepsilon^2 A^2}}_{V^2} = \frac{1}{2}CV^2$$

d) Energy of battery $= I\cdot V\cdot t$ (from $P = IV$)

$$= 10.10.3600 = 3.6\times10^5\,J$$

If cap is at 10 V, $\frac{1}{2}C(10^2) = 3.6\times10^5$

$$C = 7.2\times10^3\,F$$

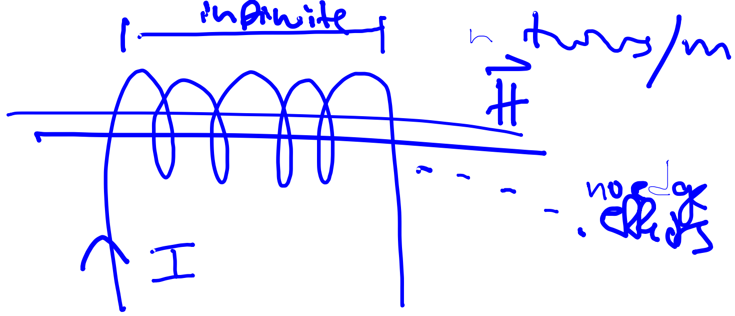
$$C = \frac{\varepsilon A}{d} = 8.85\times10^{-12}\times A\times\frac{1}{10^{-6}} = 7.2\times10^3$$

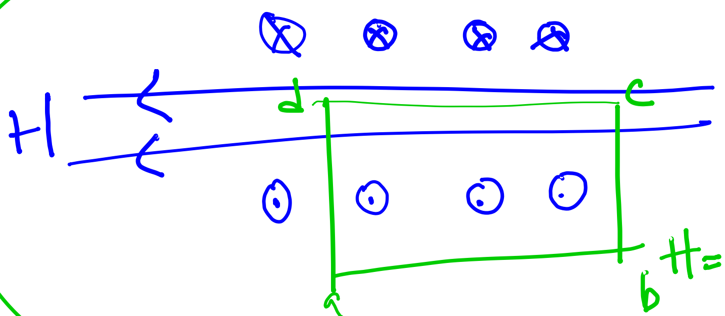
$\Rightarrow A \approx 10^9\,m^2$ (large)

if sides were 10^{-2} m, would need 10^{11} plates, stack height $10^{11}\times10^{-6} = 10^5$

6.3 a) $\int_S \vec{J}\, dA = \oint H\, dl$

from eq. 6.85





$H \cdot \text{distance}_{cd} = HL$

current thru loop is $\underbrace{n \cdot L}_{\text{number of wires in length } L} \times I$

$\therefore nLI = HL \Rightarrow H = nI\mu$

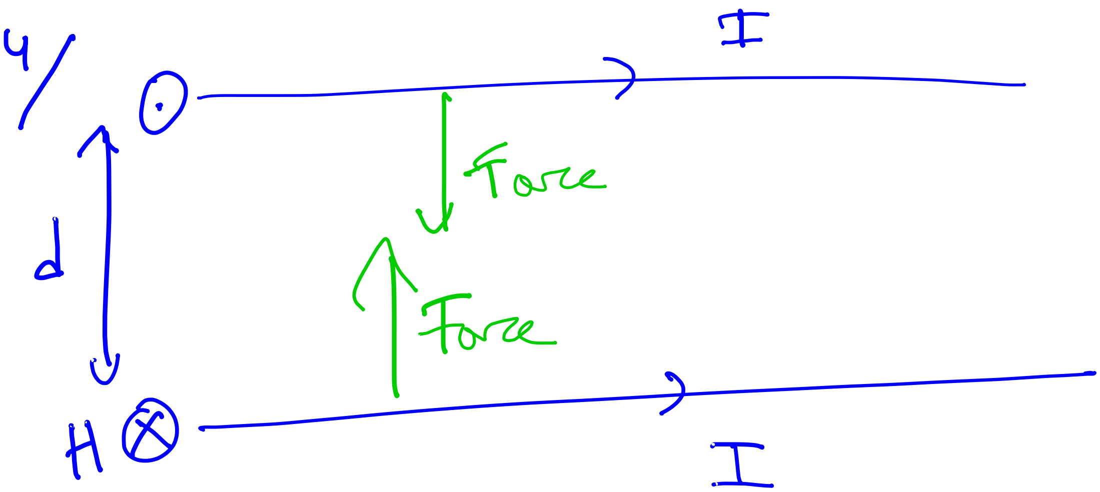
b) $U = \int \vec{B}\cdot\vec{H}\, dV = \mu\int H^2\, dV$

$= \frac{\mu}{2}\int (nI)^2\, dV = \frac{\mu}{2}(nI)^2\, \pi r^2 L$

c) $F = \frac{\partial U}{\partial r} = \frac{\mu_0(\mu_0^2 nI)^2\, \pi r L}{\mu_0 2}$

$= \frac{H^2}{\mu_0}\pi r L \approx 10^8\ N$

6.4/



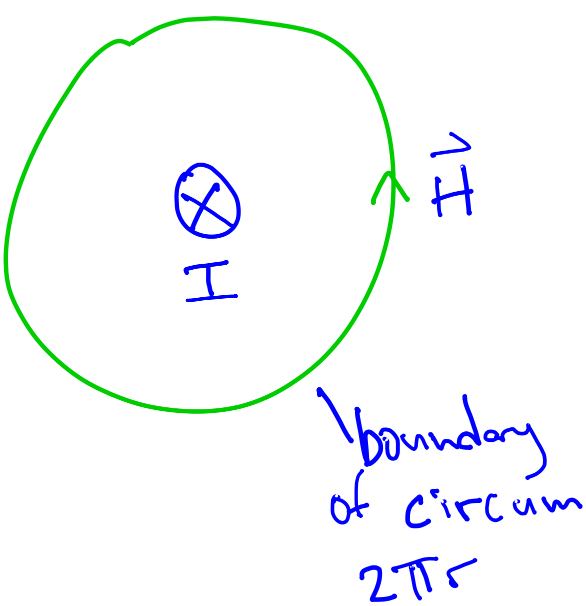
Field from one wire d m away is $d\vec{H} = \dfrac{I\, d\vec{l} \times \hat{r}}{4\pi r^2}$

$$= \frac{I}{4\pi r^2}$$

Force $d\vec{F} = I\, d\vec{l} \times \vec{B}$

$$F = \frac{I^2}{4\pi r^2} \quad ?$$

Stokes law:



$$\underbrace{\oint H\, dl}_{\vec{H}\cdot 2\pi r} = \underbrace{\int_S J\, dA}_{I}$$

current density × A = I

$$\vec{H} \cdot 2\pi r = I$$

$$\Rightarrow H = \frac{I}{2\pi r}$$

$$dF = d\vec{l} \times B = \frac{I^2}{2\pi r^2} dl \approx 10^{-7}\, N$$

1 N is from 100g of mass at 1g $\approx 10^{-7} \times 10^{-1} = 10^{-8}$ kgF

$= 10^{-5}$ gF

$= 10$ microgram F

6.5 a)

$$\vec{P} = \vec{E} \times \vec{H}$$

$$= \vec{E} \times \left(\frac{\varepsilon_0}{\mu_0} \hat{k} \times \vec{E}\right)$$

$$\langle \vec{P} \rangle = \left\langle E^2 \left(\frac{\varepsilon_0}{\mu_0}\right)^2 \right\rangle = \underbrace{\frac{1}{2}}_{\int \sin^2 = \frac{1}{2}} \underbrace{\sqrt{\frac{\varepsilon_0}{\mu_0}}}_{\approx \frac{1}{377}} E_0^2$$

$$\therefore 10^{-3} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2$$

$$\Rightarrow E_0 = 868 \frac{V}{m}$$ (pretty large ...)

does magnetic field cause effect too?

b) 1 W focused to 1mm^2 $\Rightarrow$ power is $\frac{1}{(10^{-3})^2} = 10^6$ W

$$E_{1mm} = 868 \times (10^6)^{1/2} \approx 10^4 \frac{V}{m}$$

$$E_{1\,\mu m} = 868 \times \left(10^{12}\right)^{1/2} \approx 10^{7}\ \mathrm{V/m}$$