16.1/ Ehrenfest's theorem comes from

$$i\hbar \frac{d\langle \hat{A} \rangle}{dt} = i\hbar \frac{d}{dt} \langle \psi | A | \psi \rangle$$

$$= i\hbar \left(\langle \frac{d}{dt}\psi | A | \psi \rangle + \langle \psi | \frac{dA}{dt} | \psi \rangle + \langle \psi | A | \frac{d\psi}{dt} \rangle \right)$$

Since $i\hbar \frac{\partial}{dt}|\psi\rangle = \hat{H}|\psi\rangle$,

$$= \langle \hat{H}\psi | \hat{A} | \psi \rangle + \ldots \text{ same } + \langle \psi | A | \hat{H}\psi \rangle$$

$$= \langle \psi | [\hat{H}, \hat{-}] | \psi \rangle + \langle \frac{d\hat{A}}{dt} \rangle$$

16.2/ $\mathrm{Tr}(\hat{\rho}^2) \leq 1$

$$\mathrm{Tr}(\hat{\rho}^2) \underset{\text{defn}}{=} \sum_n \langle n | \hat{\rho}^2 | n \rangle$$

Now $\hat{\rho} = \sum_i p_i |\psi_i\rangle\langle\psi_i|$

$$\mathrm{Tr}(\hat{\rho}^2) = \sum_n \langle n | \underbrace{\overbrace{\sum_{n_i} P_{n_i} | n_i \rangle}^{P_n} \langle n_i |}_{\hat{\rho}} \underbrace{\overbrace{\sum_{n_j} P_{n_i} | n_j \rangle \langle n_j |}^{P_n} n \rangle}_{\hat{\rho}}$$

$$= \sum_n P_n^2$$

16.3/

a) $\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

eigenvector is one $\quad \hat{\sigma}_x \cdot \vec{v} = \lambda v$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = -1 \cdot \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$