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Alberto DOMINGO and Carlos LAZARO (eds.)

Geometric and Structural Design of Foldable Structures

M. Babaei*, E. Sanaei[a]
* Islamic Azad University, Zanjan Branch, Iran
16, Mostashiri, Saadi Vasat, Zanjan, Iran
mbabaei@iust.ac.ir

[a] Iran University of Science and Technology, Tehran, Iran

Abstract
This paper initially investigates the previous works on foldable structures. Subsequently, generation and geometric (architectural) design of compatible foldable structures with scissor-like elements is formulated for various shapes of barrels with no geometric limitations (free forms) for the purpose of configuration processing. Special cases for configuration processing based on given formulation are studied to obtain different geometries. For example, the division of the sum of the internal angles of duplets leads to different geometries. The method employed for this division could be equal between duplets, according to arithmetic or geometric progression, or using algebraic equations. These methods are used to divide the sum of internal angles and radius of curvature; Corresponding geometries are then created and compared.

The method to generate a geometry imposed by architectural requirements is also proposed in this work. Using such ordering, one can create and model free form foldable structures. To provide changeability for geometry of the structure, a special mid-connection (pivot) is proposed and a prototype model is constructed to demonstrate the efficiency. To construct real scale foldable structures, some connections and a simple method to analyze and design of this type of connections are proposed. A graph of maximum displacement vs. height of the structure is illustrated. A design-construct methodology for foldable structures is proposed.

Keywords: foldable structure, optimum geometry, free form, scissor-like, changeable geometry, nonlinear analysis.

1. Introduction
The need for a deplorable/foldable structure has existed for many years; this appeared ever since people came across the necessity of moving from a place to another to find more appropriate environments to live in. Lightweight, small and compact structures such as tents, teepees and yurts always seemed more satisfactory to them. Therefore, it can be seen that the concept of foldable/deployable structures is not new. Among the many studies of

structures, the great Renaissance thinker and artist, Leonardo da Vinci (1452-1519) has a sketch of a simple planar deployable mechanism. Three-dimensional structures of this type were first developed by Spanish engineer Emilio Perez Pinero (1936-1972) as in Chilton [2]. The work of Pinero was sadly curtailed due to his untimely death. However, in later years his ideas were taken up and further developed by (among others) Ziegler, Calatrava, Valcarcel [11], Escrig [3-4], Hernandez, Gantes et al [5-6] , Rosenfeld [8], Logcher, Connor, Shan [9-10].

1.1. Applications

Foldable structures have two different applications, for terrestrial uses and also for outer space purposes. Some important applications are as follows:

Emergency shelters, bridges, hospitals and other public buildings; buildings and shelters in remote sites; temporary protective covers; covers for sport facilities; replaceable warehouses; hot/increment weather shelters; lightweight camping and recreational structures; exhibition structures; temporary partitions; antennas and towers; greenhouses and other agricultural applications; grand stands; travelling theatres and concerts; the packaging industry; the toy industry; other agricultural, military, social, cultural, recreational, industrial and space applications.

1.2. Advantages

These types of structures have many advantages. Some important benefits with regard to this paper are:

Speed of erection; ease of erection and prefabrication; ease of transportation and storage; reusability; Minimal skill requirements for erection and relocation; reasonable cost; simplicity of connections; changeability of geometry of structure; possibility to map (match) the structure to any shape; lightweight and packed in the deployed configuration.

2. Geometric formulation

Generally, foldable structures with scissor-like elements are classified into two groups as Compatible and Incompatible structures as in Shan [9].

In compatible structures, there is no stress and residual strain in folded state, during deployment and deployed state. These types of structures behave as mechanisms in all states, so it is essential to add other elements for stabling the structure.

In incompatible structures there is no stress in the folded state, but during deployment and in the deployed configuration, residual stress and curved members are developed. Therefore, there is no need to add other elements for stabling the structures. Also snap-through phenomenon is occurred during deployment process. This type is investigated by Gantes et al [5-6]. In this paper compatible foldable structures with scissor-like elements are investigated.

A scissor-like element, called duplet, consists of two elements, named uniplets. In general, there are two types of duplets, regular and irregular duplets as illustrated in Figure 1 in the same plane. Regular duplets are rectangular and the irregular ones are trapezoidal. Using trapezoidal duplets in two directions results in a dome, and the rectangular ones in two directions results in a flat structure. A barrel vault consists of regular, in Y direction, and irregular, in X direction, as illustrated in Figure 2. To define this type of structures, there are some parameters as shown in Figure 2. Irregular duplets with optional curvature are illustrated in Figure 3 as in Kaveh and Babaei [7]. The relationship between angles for each duplet is as follows:

$$\alpha_i = \sum_{j=0}^{i-1} b_j + b_i / 2 \tag{1}$$

$$\beta_i = \pi / 2 - b_i / 2 \tag{2}$$

And the deployability condition is as:

$$O_1A + O_1B = O_2A + O_2B \tag{3}$$

By changing the value of α for neighbor duplets, any curved shapes could be developed. The same value for r, results in a circular curvature barrel vault as Figure 2. In any way, three parameters of l, h, b or r must be assumed; then the last parameter will be obtained (one equation, four unknown parameters). Considering parameters l, h and b as known, the value of r could be calculated by satisfying the deployability conditions as follows:

$$r = \frac{\sqrt{l^2 - h^2 \cos^2(b/2)}}{2\sin(b/2)} - \frac{h}{2} \tag{4}$$

Where r is the radius of curve, l is the uniplet length, h is the structural depth and b is the internal angel.

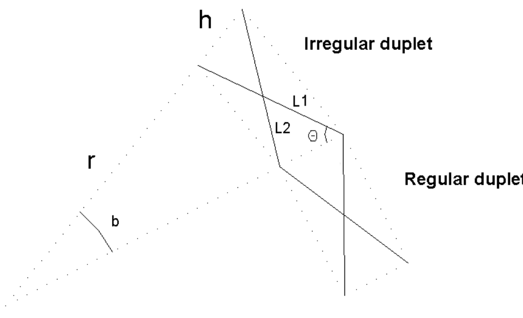

Figure 1: Regular and irregular duplets

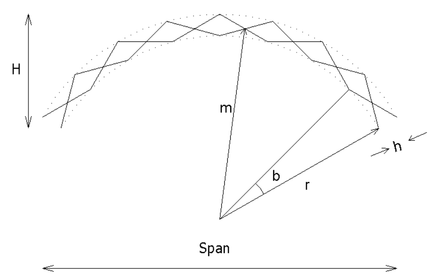

Figure 2: Barrel vault with circular curvature

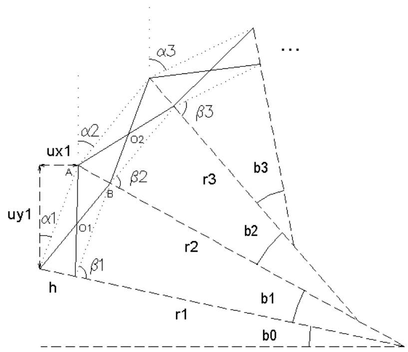

Figure 3: Trapezoidal duplets with arbitrary curvatures with same length of elements

And other parameters are obtained as:

$$\theta = \sin^{-1}\left(\frac{r}{l}\sin b\right) \tag{5}$$

$$m = (r+h)\frac{\sin\theta}{\sin(\theta + b/2)} \tag{6}$$

$$L_1 = (r+h)\frac{\sin(b/2)}{\sin(\theta + b/2)} \tag{7}$$

$$L_2 = l - L_1 \tag{8}$$

Total span and height of structure as shown in Figure 3 are:

$$Total\ span = \sum ux = \sum_{i=1}^{n}[2(r_i + h)\sin(b_i/2)\sin\alpha_i] \tag{9}$$

$$Total\ height = \sum uy = \sum_{i=1}^{n}[2(r_i + h)\sin(b_i/2)\cos\alpha_i] \tag{10}$$

3. Case studies

For the case when the sum of internal angles for duplets is known ($S = \sum b_i$), division of this sum leads to different geometries, but some other parameters as number and the length of duplets are fixed. Other parameters can be obtained using Equations (4-10). One can also use half of S for symmetric structures. On the other hand, one can divide and generate other geometries by applying this method to other parameters as radius of curvature. In this paper division of sum of the internal angles and radius of curvature are investigated. It should be noted that, one can use the length of the elements as variable parameter, when deciding to build foldable structures using available materials and sections.

3.1. Dividing internal angle

In this case, r, c and h are fixed and the sum of internal angles is considered to be 90^o for a symmetrical structure. This angle is initially divided equally between duplets, so the produced geometry will be half of a circle as shown in Figure 4. If the internal angle is divided according to an arithmetic progression, then the relating geometry will be as shown in Figure 5. Using geometric progression, the produced geometry will be as Figure 6. Figure 7 illustrates the geometry obtained using algebraic equations. All figures are illustrated in the same scale such that one can compare the differences.

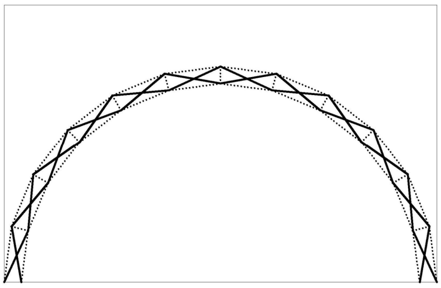

Figure 4: equally dividing

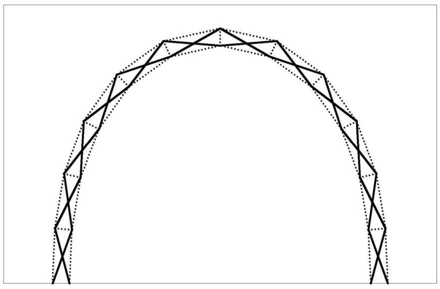

Figure 5: arithmetic progression

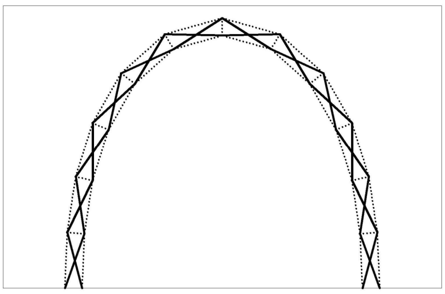

Figure 6: geometric progression

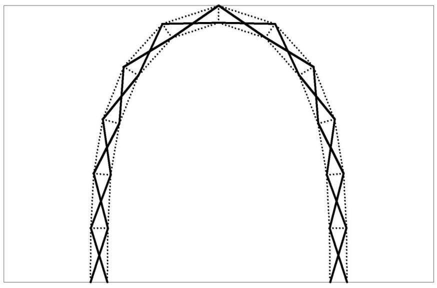

Figure 7: algebraic equation

3.2. Changing radius of curvature

Other types of geometries could be obtained by changing the radius. In this case, b, c and h are fixed and the radius of curvature is considered to have variable parameter. Assigning equal amounts for the radius, the produced geometry will be an arch of a circle as shown in Figure 8. Using arithmetic progression, geometric progression and algebraic equations, arched geometries are obtained as illustrated in Figures 9 to 11. As shown in these Figures, the sum of the internal angels is not 180^o and in not the same for all cases. A duplet satisfying some conditions as deployability conditions must be added in the apex of the produced geometry to obtain a symmetric structure since the main produced geometry will not be symmetric.

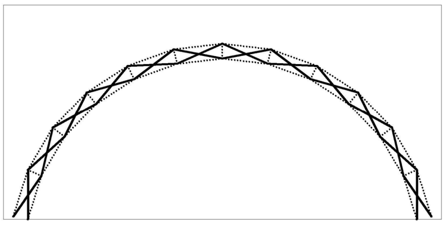

Figure 8: equal radius of curvature

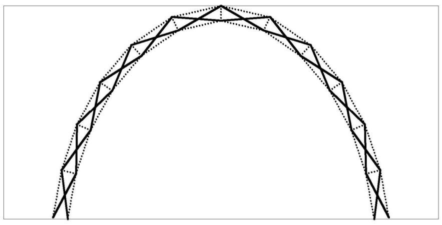

Figure 9: arithmetic progression

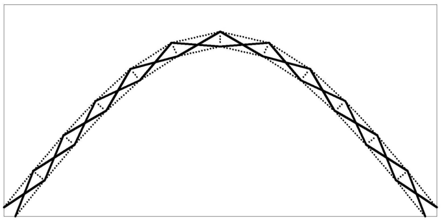

Figure 10: geometric progression

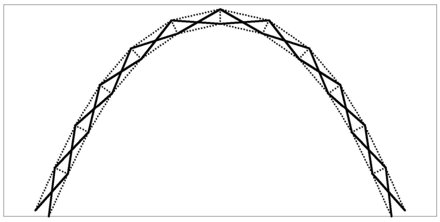

Figure 11: algebraic equation

3.3. Matching a structure to special geometry

In some cases, the architecture of the structure may impose the geometry of the structure. In such a case, one can select an appropriate or arbitrary geometry for foldable structure (i.e. free form structure). What is important here is that how one can map (match) a foldable structure to the specified curvature. It is feasible using formulation proposed in this paper. As an example, if the geometry is to be considered as half of an ellipse the relating geometry after mapping will be as illustrated in Figure 12.

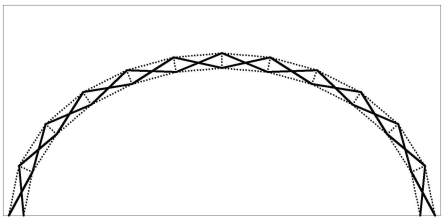

Figure 12: Semi ellipse foldable structure

4. Geometric changeability and sample connections

Generally, two types of connections are used to construct foldable structures: the end of element connection; and the middle connection (pivot). At the end of each element it is needed to provide a hinge, so that all the elements could rotate and the folding and deploying process is made possible. Based on morphology of the structure, the end connection could be one of those illustrated in Figure 13. If up to four elements are connected together perpendicularly, the connection illustrated in Figure 13 (left) is applicable. Figure 13 (right) displays a connection that provide hinge if more than four elements are connected or when they are not perpendicular.

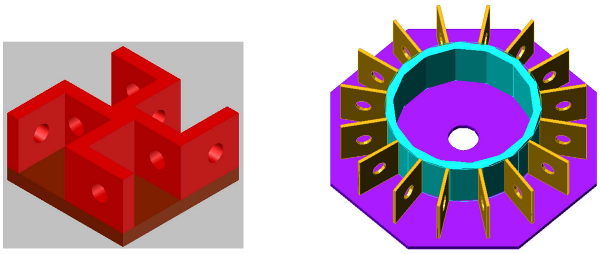

Figure 13: End connections; left: for perpendicular elements, right: for free form

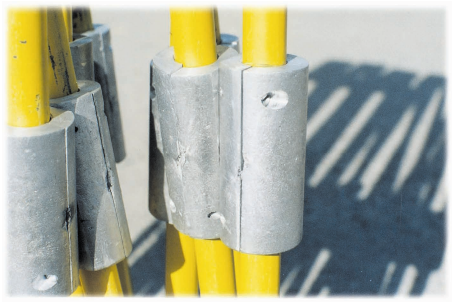

Figure 14: Special connection for changeable foldable structure

Figure 15: Flat foldable structure in the folded and fully deployed configurations

Figure 16: Barrel vault foldable structure

The second type of connection used in a foldable structure is the middle one, in such way that two elements are pivoted together. Providing such pivot is possible using bolt or rivet. However, in this case, the geometry of the structure is the same at all times while it is deployed. The location of this pivot must be replaced by others to change the geometry. In other words, it needs to provide other holes so the elements become weak as a result of these holes. To provide changeability for the geometry of the structure without weakening the elements, scissor-like covering connections as illustrated in Figure 14 is proposed by Babaei [1]. Changing the location of this connection along with the element using formulation mentioned before, changeability of the structure in the deployed configuration will be provided. Figure 15 illustrates a flat structure in the folded and fully deployed configurations. If the locations of the connections are changed, other geometries will be produced similar to the barrel vault illustrated in Figure 16.

5. Connection analysis and Design

5.1. Simple method to analyze end connection

The axial force F is obtained by analyzing the structure, M33 equals zero (because of the hinge present) and M22 is negligible. To model the structure for analyzing and designing with the help of the analysis programs called SAP, elements are concurrent in a same point hence the axial forces are concurrent. However, in a real structure there is an eccentricity, therefore F creates a secondary moment along M22. Considering F as the horizontal component (about connection) of maximum axial force obtained by analysis and M as the secondary moment due to eccentricity of F. In a barrel, the internal forces for longitudinal regular scissor-like elements (SLEs) are negligible. Thus the internal axial forces for irregular ones are considered. An end connection is shown in Figure 17. The free body diagram of the connection is shown in Figure 17 (right). Using equilibrium, values of F1, F2 and M are obtained as:

$$F1 = \frac{2d+t}{2(d+t)} \times F \qquad (11)$$

$$F2 = \frac{t}{2(d+t)} \times F \tag{12}$$

$$M = \frac{F.d}{2} \tag{13}$$

For example, if $t = 0.1 * d$ then $F1 = 10.95 * F$ and $F2 = 0.05 * F$.

It is clear that F1 is much more than F2, so the inner walls must be thicker than the edges.

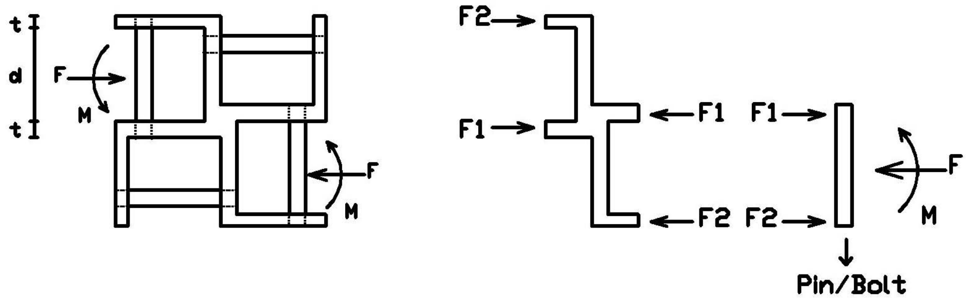

Figure 17: End connection; left: Plan view, right: Free body diagram

5.2. Connection design

Based on the output of the analysis, the critical connection seems to be the end one (next to the support). A dome or flat structure carries the load in two dimensions. In a barrel, elements on curve (i.e. irregular SLEs) bear the loads, and longitudinal elements (i.e. regular or rectangular SLEs) have small internal forces. In other words, the structure carries the load in one dimension. Therefore, to design the connection, only internal forces of the irregular SLEs should be considered. Also, for simplicity, the maximum internal forces are used to design. Figure 18 illustrates all internal forces applied to the end connection. Using maximum amount of the above results from analysis, F is obtained as:

$$F = \sqrt{P^2 + V^2} \tag{15}$$

Maximum stress in the connection is obtained as:

$$\frac{F1}{t * D} \tag{16}$$

where D is the diameter of the pin or bolt which connects the element to the connection.

Maximum squeezing stress in the section of the uniplet (approximately) equals:

$$\frac{F1}{tw * D} \tag{17}$$

To design the middle connection (pivot), maximum reaction of the pin with the help of analysis results in as:

$$R = \sqrt{(R_x)^2 + (R_y)^2} \tag{18}$$

where R_x and R_y are the horizontal and vertical components of shear force of pin, respectively.

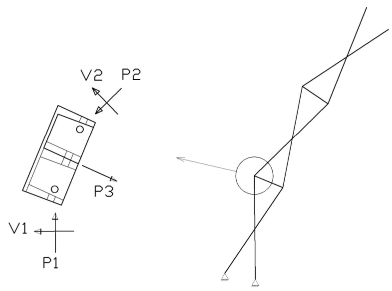

Figure 18: Applied forces for connection

The process of design, construct and erect of foldable structures is shown in Figure 19.

6. Concluding remark

In this paper the formulation of foldable structures with free form for geometric design is developed. Different geometries using algebra are illustrated to demonstrate the effect of algebra in geometry. To construct real scale free form foldable structures, some connections and a simple method to analyze and design of this type of connections are proposed. Prototype models and connections show the efficiency and accuracy of the formulation. An applicable flowchart to design, construct and erect the foldable structures is proposed. This methodology develops design-construct process for free form foldable structures.

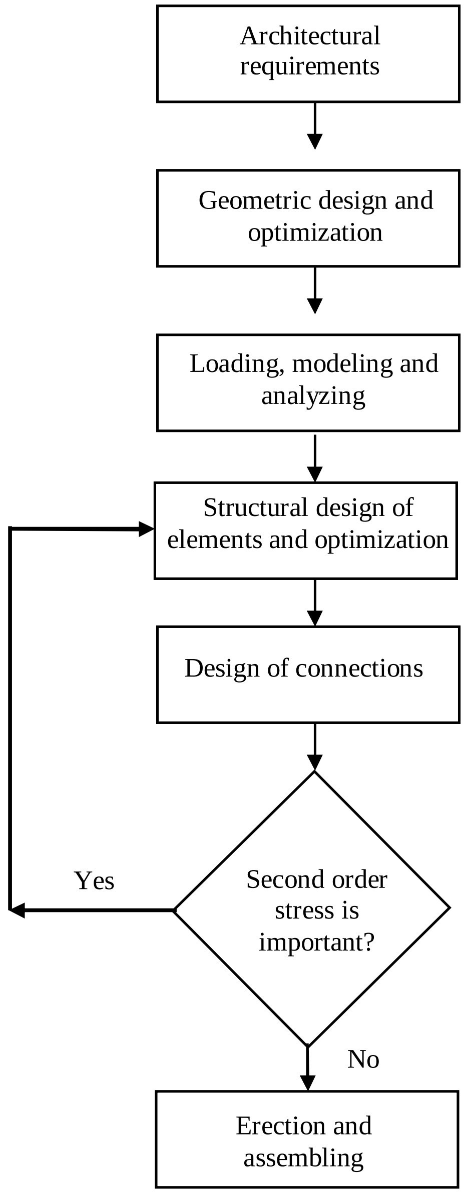

Figure 19: Design-construct process of foldable structures

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