“Finding” a Pulse Shape

Jason Taylor
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“The laser cavity finds the pulse that minimizes loss--it's like magic.”
My Research

I investigate the usefulness of laser cavities and laser dynamics for information processing.

Kerr Lens Mode-Locked laser cavities
Project Goals

• Discover what a KLM laser minimizes
  – Cavity loss?
  – Population Inversion?

• Use results to predict good bit representation and/or logical operators
  – Space
  – Time
  – Phase
  – Power
Kerr-Lens Mode Locked Oscillator
The Equation

\[ \frac{1}{T_R} \frac{\delta}{\delta t} a = (g-l) a + \left( \frac{1}{\Omega_f^2} + jD \right) \frac{\delta^2}{\delta t^2} a + (\gamma - j\delta) |a|^2 a \]

This equation describes the evolution of a short pulse over one round trip in a KLM cavity.
Mode Locking

\[ \omega_0 + 4\Delta \omega_{ax} \]
\[ \omega_0 + 3\Delta \omega_{ax} \]
\[ \omega_0 + 2\Delta \omega_{ax} \]
\[ \omega_0 + \Delta \omega_{ax} \]
\[ \omega_0 \]
\[ \omega_0 - \Delta \omega_{ax} \]
\[ \omega_0 - 2\Delta \omega_{ax} \]
\[ \omega_0 - 3\Delta \omega_{ax} \]
\[ \omega_0 - 4\Delta \omega_{ax} \]
\[ \omega_0 - 5\Delta \omega_{ax} \]

\[ t_{RT} \]
Artificial Fast Saturable Absorbers

\[ n(I) = n + n_2 I \]
\[ n_2 = \frac{2\eta_0}{n^2\varepsilon_0}\chi^{(3)} \]

fast saturable absorber

\[ s(t) = \frac{s_0}{1 + I(t)/I_{sat}} \]

[Hau00]
fast saturable absorber

\[ s(t) = \frac{s_0}{1 + I(t)/I_{\text{sat}}} \]

master equation:

\[
\frac{1}{T_R} \frac{\delta}{\delta t} a = (g - l)a + \left( \frac{g}{\Omega_g^2} + \frac{1}{\Omega_f^2} \right) \frac{\delta^2}{\delta t^2} a + \gamma |a|^2 a
\]

\[ a_0(t) = A_0 \text{sech}(t/\tau) \]

solution unbounded

Siegman, Haus [Hau00]
Kerr-Lens Mode Locked Laser

Ti:Sapphire KLM oscillators are available commercially—where else would this graphic come from?
Soliton Effects in Ultrashort Pulses

\[ \Delta n = n_2 I(t) \]

Self Phase Modulation (SPM)

\[ \Delta a = -j \delta |a|^2 a \]

Group Velocity Dispersion (GVD)

\[ \Delta a = jD \frac{d^2}{dt^2} a \]

New master equation with GVD and SPM:

\[ \frac{1}{TR} \frac{\delta}{\delta T} a = \left(g-l \right) a + \left(1 + jD \right) \frac{\delta^2}{\delta t^2} a + \left(\gamma-j\delta\right) |a|^2 a \]

\[ \text{GVD} \]

\[ \text{SPM} \]

\[ \delta_0 = \text{no SPM} \]

\[ D_n = \text{GVD} \]

\[ \delta_n = 0 \]

\[ \delta_n = 2 \]

\[ \delta_n = 4 \]

\[ \tau/\tau_0 \]

pulse width
Haus’ Master Equation

master equation with GVD and SPM:

\[
\frac{1}{T_R} \int_{\delta T} \frac{\delta}{\delta T} a = (g-l)a + \left( \frac{1}{\Omega_f^2} + jD \right) \frac{\delta^2}{\delta t^2} a + (\gamma - j\delta) |a|^2 a
\]

Gain depletion

\[
g = g(T) = \frac{g_0}{1 + \frac{E_P(T)}{E_{sat}}}
\]

where

\[
E_P(T) = \int_{-\infty}^{+\infty} |A(T, t)|^2 dt
\]

\[
E_P(T) = \Delta t \sum_{n=0}^{N} |A(T, n)|^2
\]
Numerical Simulation

Here we look at the equation properties via numerical simulation.
Separate into Linear and Non-Linear Operators

\[
\frac{1}{T_R} \frac{\delta}{\delta T} a = (g - l) a + \left( \frac{1}{\Omega_f^2} + jD \right) \frac{\delta^2}{\delta t^2} a + (\gamma - j\delta) |a|^2 a
\]

\[
\frac{\partial A}{\partial T} = (\hat{D} + \hat{N}) A
\]

where

\[
\hat{D} = g - l + (D_{gf} + jD) \frac{\partial^2}{\partial t^2}
\]

\[
\hat{N} = (\gamma - j\delta) |A|^2
\]

\[
A(T + \kappa, t) = \exp[\kappa(\hat{D} + \hat{N})] A(T, t)
\]
Simulation

Linear Terms
\[ \hat{D} \bigg/ 2 \]

Iterative Non-linear Step
\[ \hat{N} \]

Update gain and bandwidth filter parameters

Linear Terms
\[ \hat{D} \bigg/ 2 \]
Gain, SAM, no GVD, no SPM

pulse width

frequency
Gain, SAM, GVD, and SPM

Pulse width

Frequency
What is minimized?

- Cavity loss?
- Gain medium population inversion?
- Pulse width?
Haus’ Master Equation

master equation with GVD and SPM:

\[
\frac{1}{T_R} \frac{\delta}{\delta T} a = (g-l)a + \left( \frac{1}{\Omega_f^2} + jD \right) \frac{\delta^2}{\delta t^2} a + (\gamma-j\delta)|a|^2 a
\]

Gain depletion

\[
g = g(T') = \frac{g_0}{1 + \frac{E_P(T)}{E_{sat}}}
\]

where

\[
E_P(T) = \int_{-\infty}^{+\infty} |A(T, t)|^2 dt
\]

\[
E_P(T) = \Delta t \sum_{n=0}^{N} |A(T, n)|^2
\]
Complex Ginzburg-Landau Equation

master equation with GVD and SPM:

$$\frac{1}{TR} \frac{\delta}{\delta T} a = (g-l)a + \left( \frac{1}{\Omega_f^2} + jD \right) \frac{\delta^2}{\delta t^2} a + (\gamma - j\delta)|a|^2 a$$

Gain depletion

$$g = g(T) = \frac{g_0}{1 + \frac{E_P(T)}{E_{sat}}}$$

where

$$E_P(T) = \int_{-\infty}^{+\infty} |A(T, t)|^2 dt$$

$$E_P(T) = \Delta t \sum_{n=0}^{N} |A(T, n)|^2$$
Complex Ginzburg-Landau Equation

master equation:

\[
\frac{1}{T_R} \frac{\delta}{\delta T} a = (g-l)a + \left( \frac{1}{\Omega_f^2} + jD \right) \frac{\delta^2}{\delta t^2} a + (\gamma - j\delta)|a|^2 a
\]

soliton like pulse

\[a(t) = A_0 \ \text{sech}^{(1+j\beta)}(\frac{t}{\tau})\]

CW solution

\[a(t) = A_0 \exp(-j\omega t)\]

general CGLE

\[
\frac{\partial}{\partial T} A = A + (1+jc_1) \frac{\partial^2}{\partial t^2} A - (1+jc_2)|A|^2 A
\]
Complex Ginzburg-Landau Equation

general CGLE

\[ \frac{\partial}{\partial T} A = A + (1 + jc_1) \frac{\partial^2}{\partial t^2} A - (1 + jc_2)|A|^2 A \]
Lyapunov Function

soliton like pulse

$$a(t) = A_0 \, \text{sech}^{(1+j\beta)}\left(\frac{t}{\tau}\right)$$

CW solution

$$a(t) = A_0 \, \exp(-j\omega t)$$

A good approximate Lyapunov function is known for a CW stationary solution.

No Lyapunov function is known for soliton solutions—too close to chaos. IMHO
Simplify Equation

\[ \frac{1}{T_R} \delta \frac{\partial}{\partial T} a = (g-l)a + \left( \frac{1}{\Omega_f^2} + jD \right) \frac{\delta^2}{\delta t^2} a + (\gamma - j\delta)|a|^2a \]

\[ \frac{\partial}{\partial T} A(T, t) = jD \frac{\partial^2}{\partial t^2} A - j\delta |A|^2 A \]

ignore gain depletion, BW filtering and SAM
Non-linear Schrodinger Equation

$$\frac{\partial}{\partial T} A(T, t) = jD \frac{\partial^2}{\partial t^2} A - j\delta |A|^2 A$$

Lyapunov Function:

$$V = \int_{-\infty}^{+\infty} dt \left[ -D |A|^2 + \frac{\delta}{4} |A|^4 a + \left| \frac{\partial}{\partial t} A \right|^2 \right]$$

Looks a lot like minimizing the action.
Numerical Confirmation

Try Lyapunov function in simulator.
NLSE Lyapunov function on CGLE with gain saturation
Gain, SAM, GVD, and SPM

pulse width

frequency

Gain, SAM, GVD, and SPM
Zoom in on Hump

"Lyapunov" Function

$V$ vs. cavity round trips

$V \times 10^{14}$
Zoom in on 2\textsuperscript{nd} Hump
Zoom in on 3rd Hump
Zoom in on 4th Hump
Conclusions

• We found a fractal.
• The NLSE approximate Lyapunov function isn’t valid far away from the soliton solution.
• Is a numerical stability analysis of the CGLE sufficient?