# “Finding” a Pulse Shape

Jason Taylor

May 16, 2005

“The laser cavity finds the pulse that minimizes loss--it's like magic.”

# My Research

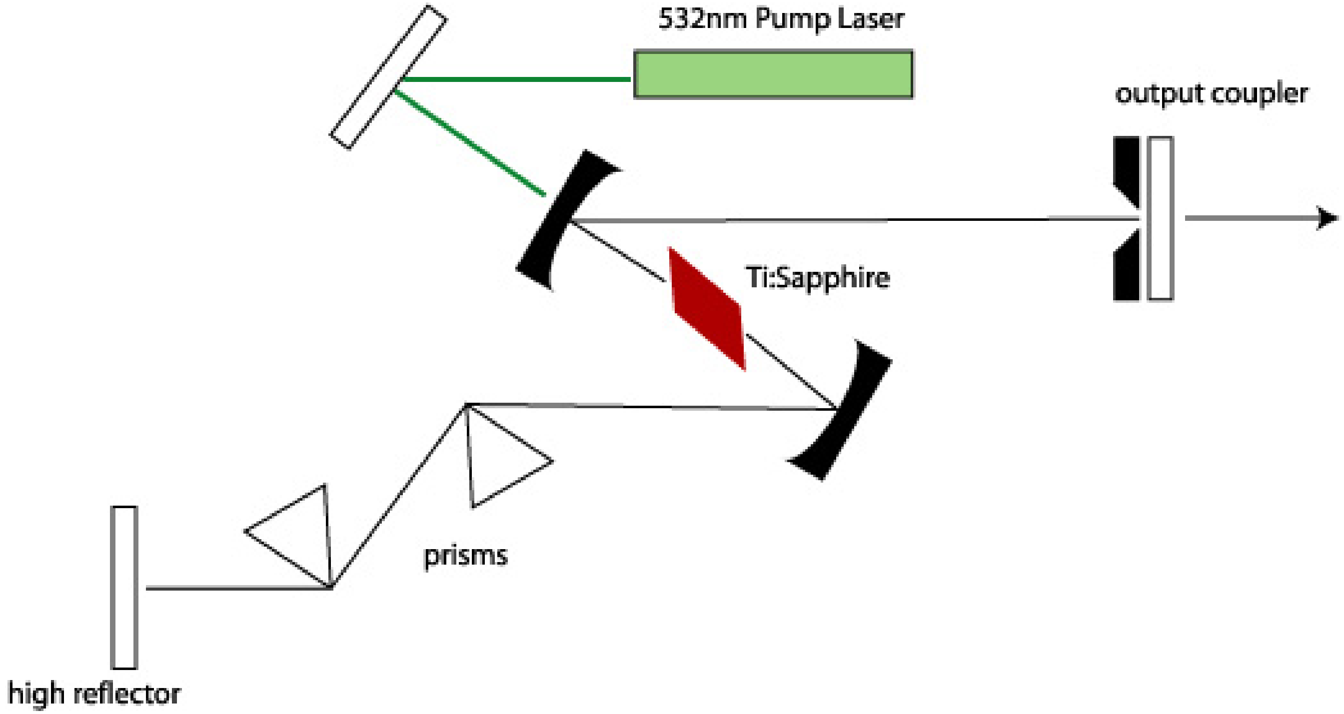
I investigate the usefulness of laser cavities and laser dynamics for information processing.

Kerr Lens Mode-Locked laser cavities

# Project Goals

- Discover what a KLM laser minimizes
  - Cavity loss?
  - Population Inversion?
- Use results to predict good bit representation and/or logical operators
  - Space
  - Time
  - Phase
  - Power

# Kerr-Lens Mode Locked Oscillator

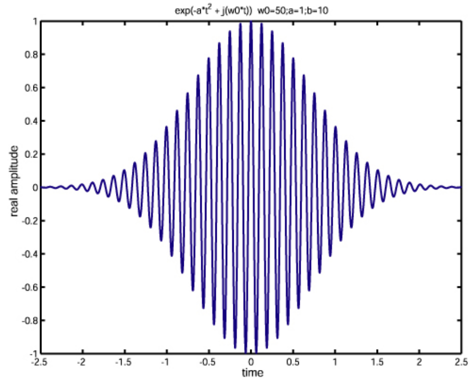


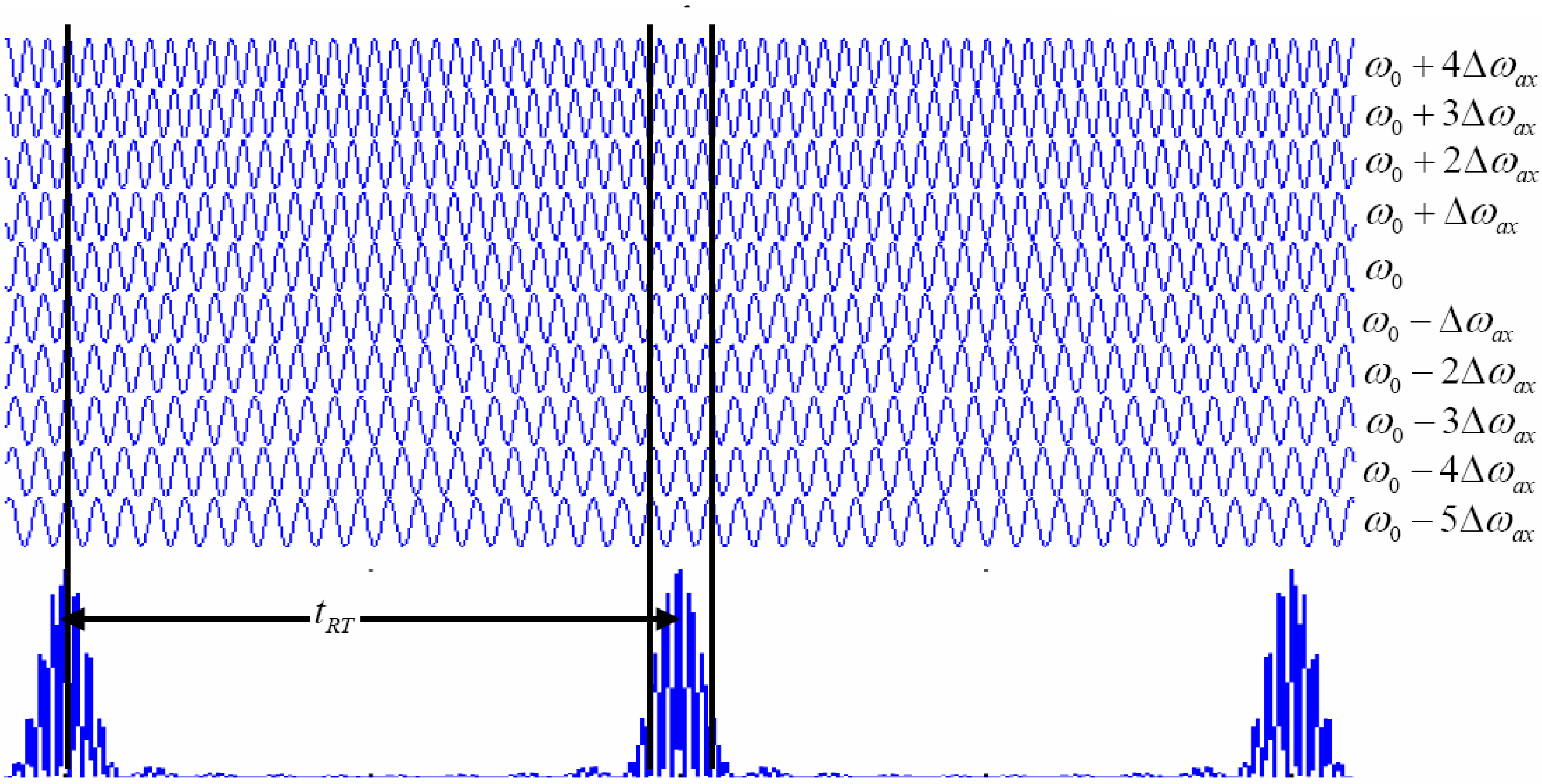
# The Equation

$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \Big(\frac{1}{\Omega_f^2} + \underbrace{jD}_{\text{GVD}}\Big)\frac{\delta^2}{\delta t^2}a + (\gamma - \underbrace{j\delta)}_{\text{SPM}}|a|^2 a$$

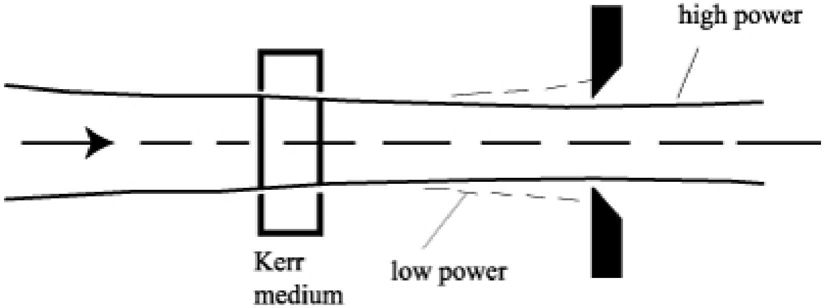
This equation describes the evolution of a short pulse over one round trip in a KLM cavity.

# Mode Locking



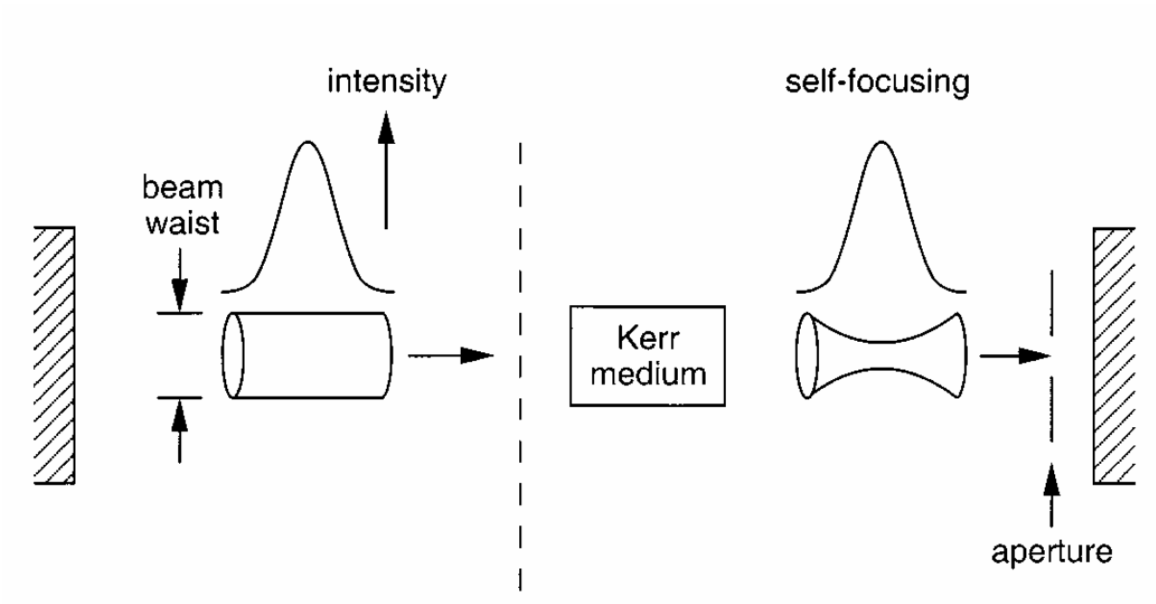



# Artificial Fast Saturable Absorbers




$$n(I) = n + n_2 I$$

$$n_2 = \frac{2\eta_0}{n^2\epsilon_0}\chi^{(3)}$$



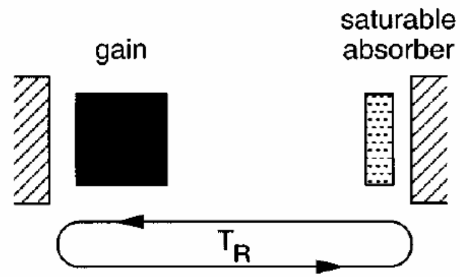

fast saturable absorber

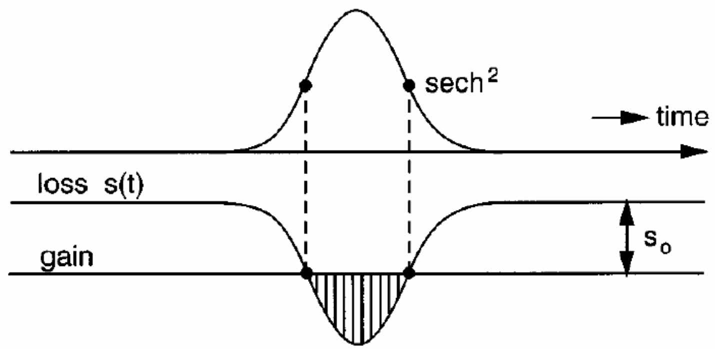
$$s(t) = \frac{s_0}{1 + I(t)/I_{\text{sat}}}$$

[Hau00]

fast saturable absorber

$$s(t) = \frac{s_0}{1 + I(t)/I_{\text{sat}}}$$




master equation:

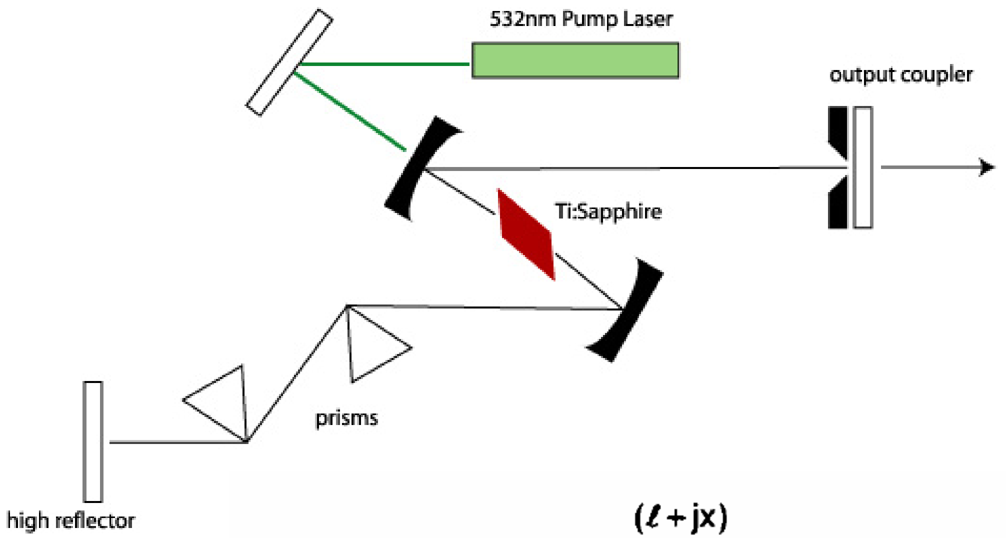
$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \left(\frac{g}{\Omega_g^2} + \frac{1}{\Omega_f^2}\right)\frac{\delta^2}{\delta t^2}a + \gamma|a|^2 a$$

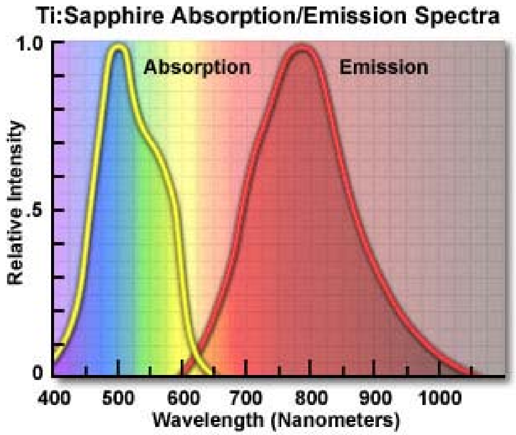
$$a_0(t) = A_0 \text{sech}(t/\tau)$$

solution unbounded

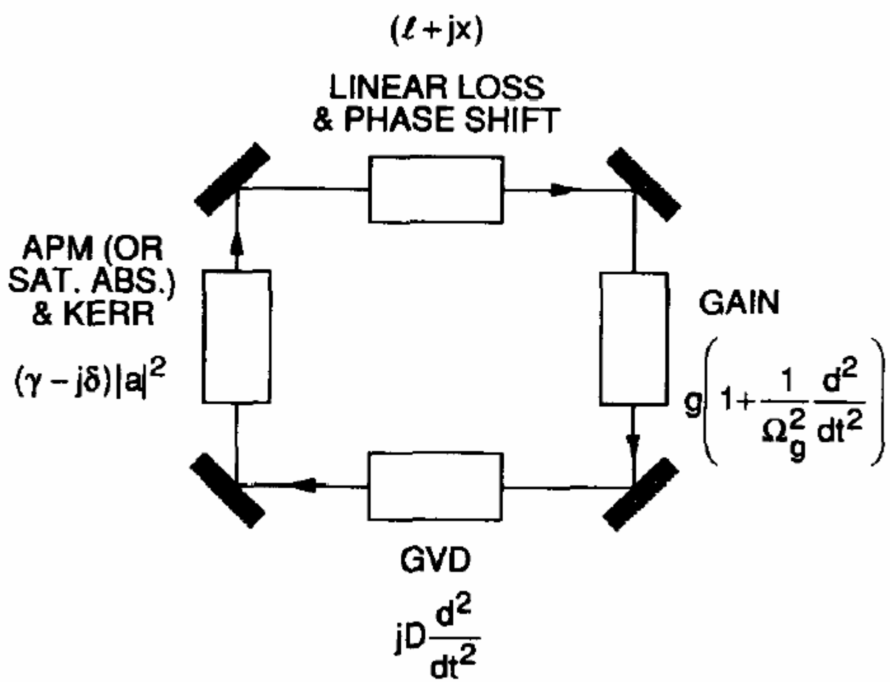
Siegman, Haus [Hau00]

# Kerr-Lens Mode Locked Laser




Ti:Sapphire KLM oscillators are available commercially—where else would this graphic come from?



# Soliton Effects in Ultrashort Pulses

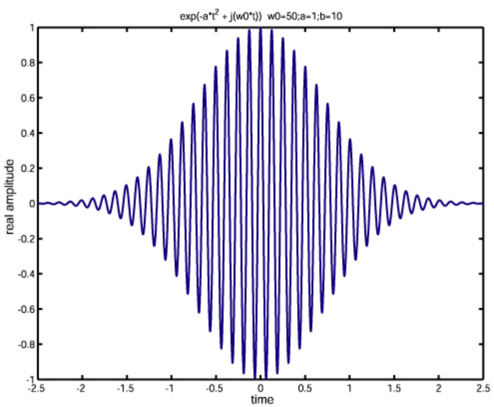
$$\Delta n = n_2 I(t)$$

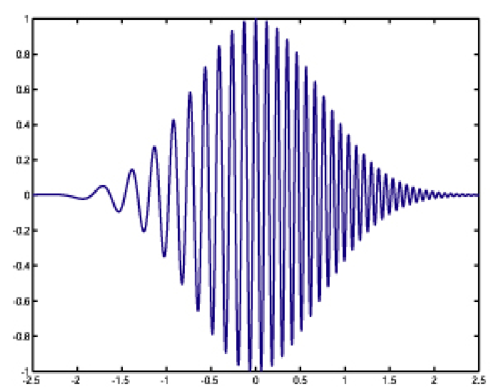
Self Phase Modulation (SPM)

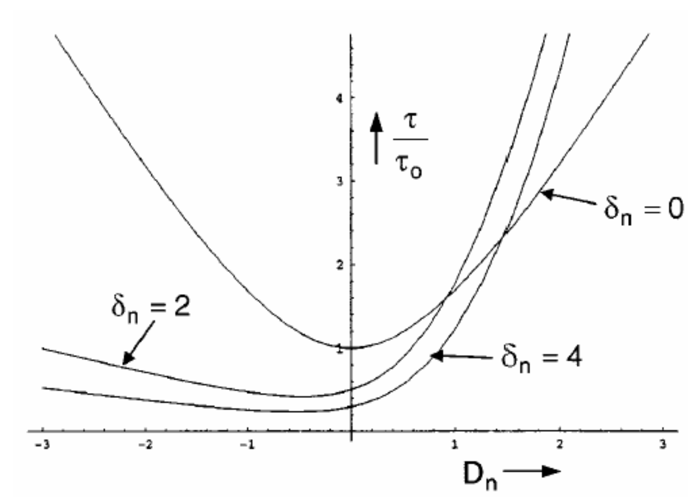
$$\Delta a = -j\delta|a|^2 a$$

Group Velocity Dispersion (GVD)

$$\Delta a = jD\frac{d^2}{dt^2}a$$






pulse width

$\delta_0$ = no SPM
$D_n$ = GVD

New master equation with GVD and SPM:

$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \Big(\frac{1}{\Omega_f^2} \underbrace{+jD}_{\text{GVD}}\Big)\frac{\delta^2}{\delta t^2}a + (\gamma \underbrace{-j\delta)|a|}_{\text{SPM}}{}^2 a$$

# Haus’ Master Equation

master equation with GVD and SPM:

$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \Big(\frac{1}{\Omega_f^2} \underbrace{+jD}_{\text{GVD}}\Big)\frac{\delta^2}{\delta t^2}a + (\gamma \underbrace{-j\delta)|a|}_{\text{SPM}}{}^2 a$$

Gain depletion

$$g = g(T) = \frac{g_0}{1 + \frac{E_P(T)}{E_{sat}}}$$

where

$$E_P(T) \;=\; \int_{-\infty}^{+\infty} |A(T,t)|^2 dt$$

$$E_P(T) \;=\; \Delta t \sum_{n=0}^{N} |A(T,n)|^2$$

# Numerical Simulation

Here we look at the equation properties via numerical simulation.

# Separate into Linear and Non-Linear Operators

$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \left(\frac{1}{\Omega_f^2} + jD\right)\frac{\delta^2}{\delta t^2}a + (\gamma - j\delta)|a|^2 a$$

$$\frac{\partial A}{\partial T} = (\hat{D} + \hat{N})A$$

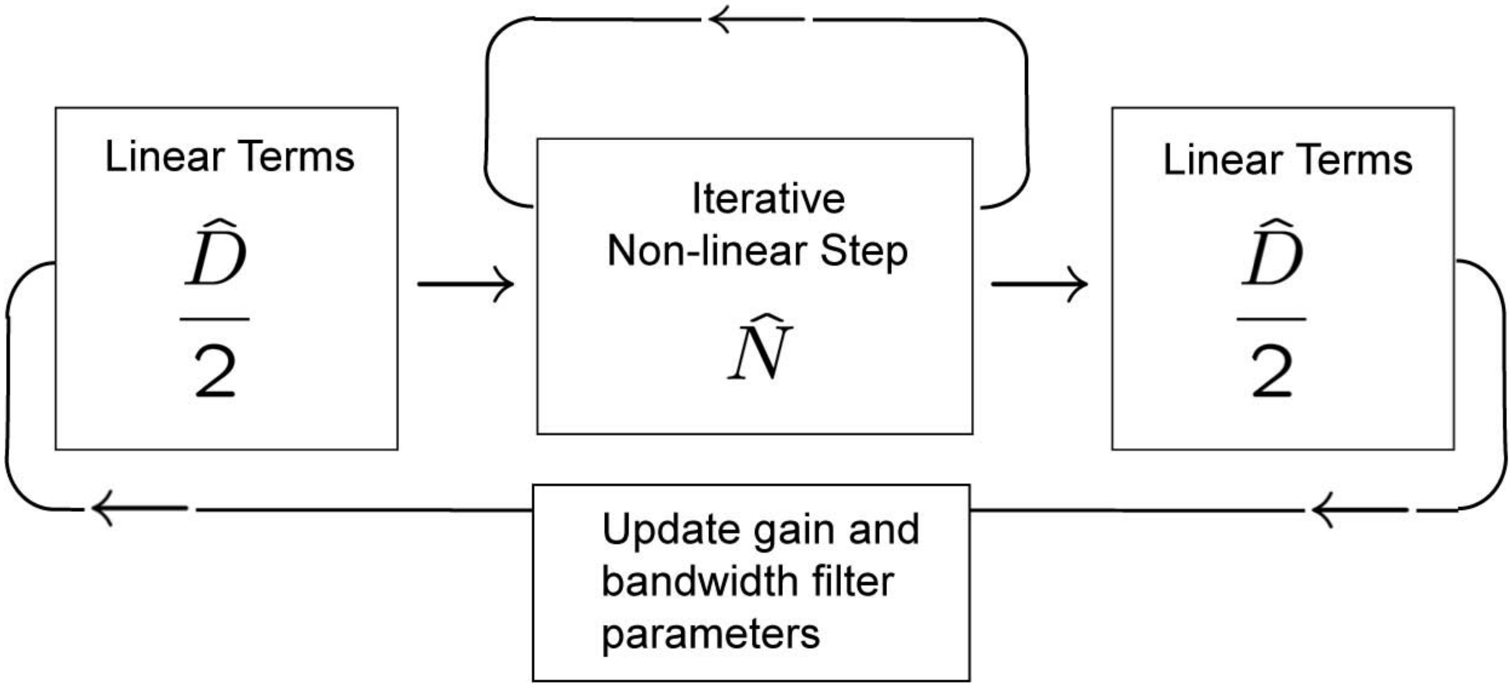
where

$$\hat{D} = g - l + (D_{gf} + jD)\frac{\partial^2}{\partial t^2}$$

$$\hat{N} = (\gamma - j\delta)|A|^2$$

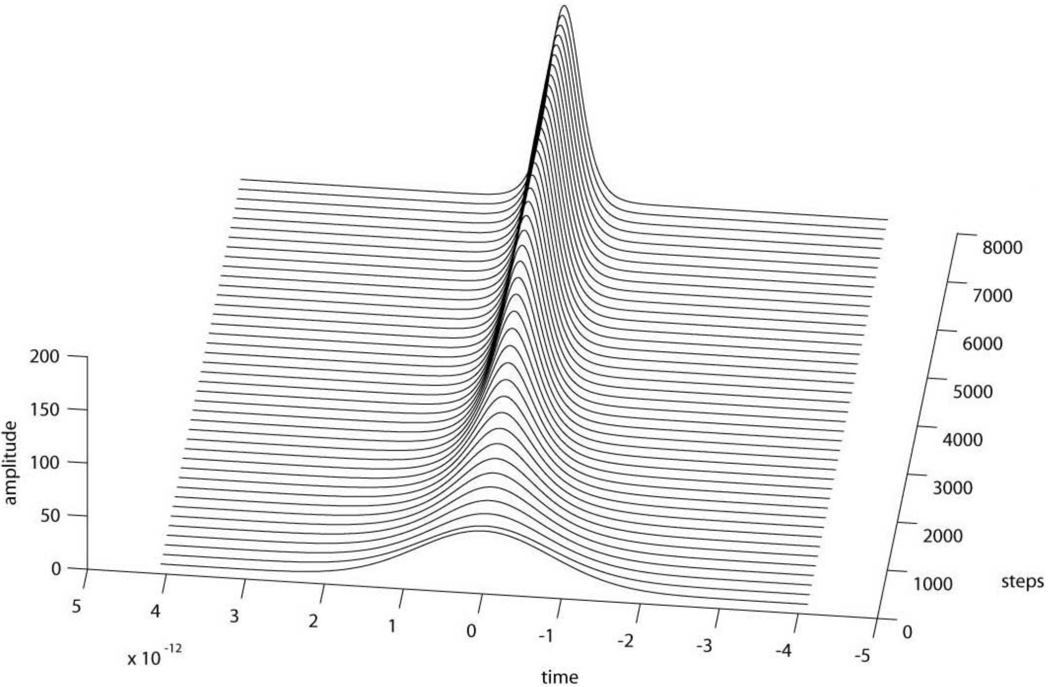
$$A(T+\kappa, t) = \exp[\kappa(\hat{D} + \hat{N})]A(T, t)$$

# Simulation

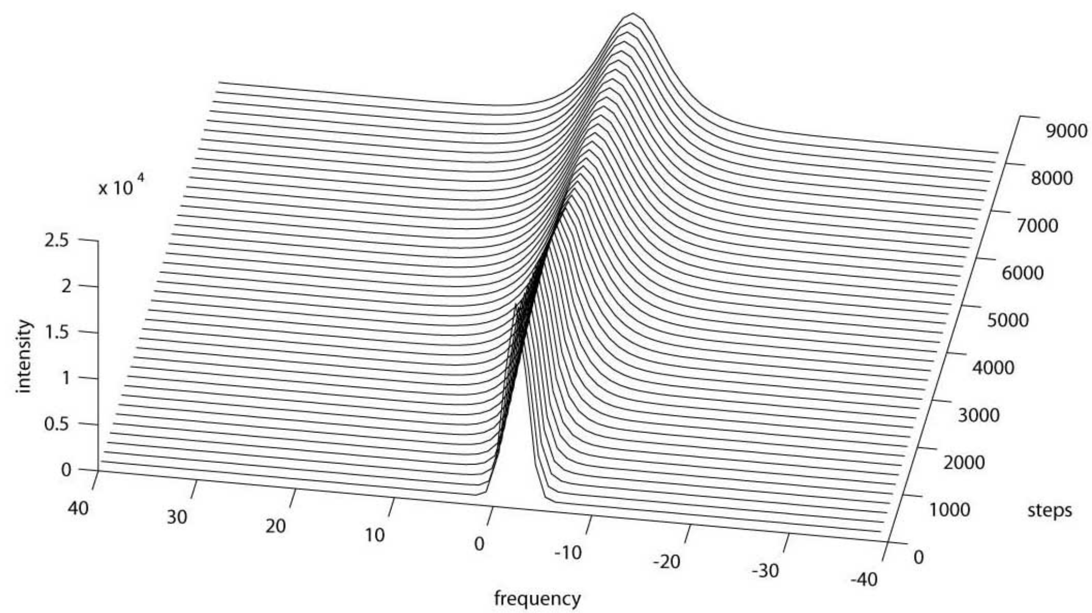


Linear Terms
$\frac{\widehat{D}}{2}$
Iterative
Non-linear Step
$\widehat{N}$
Linear Terms
$\frac{\widehat{D}}{2}$
Update gain and
bandwidth filter
parameters

# Gain, SAM, no GVD, no SPM

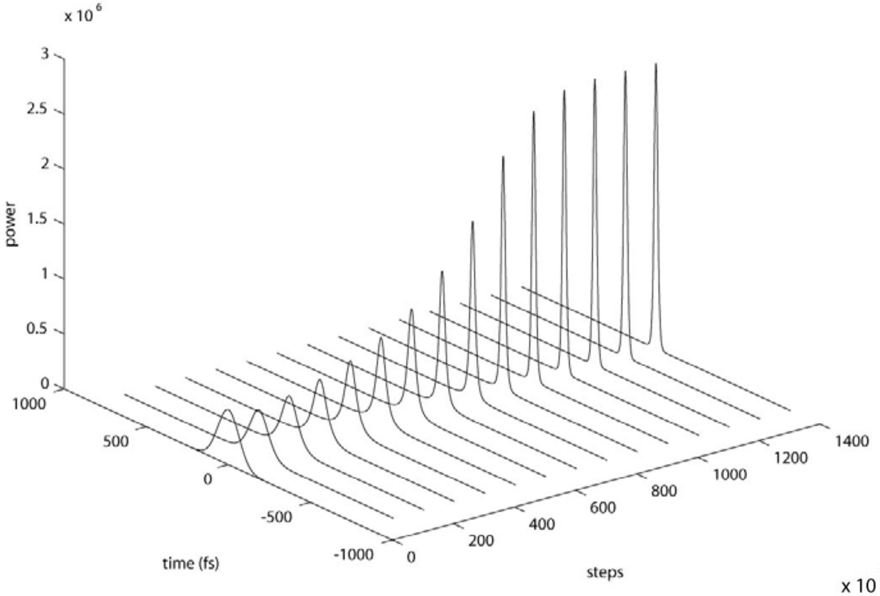



pulse width

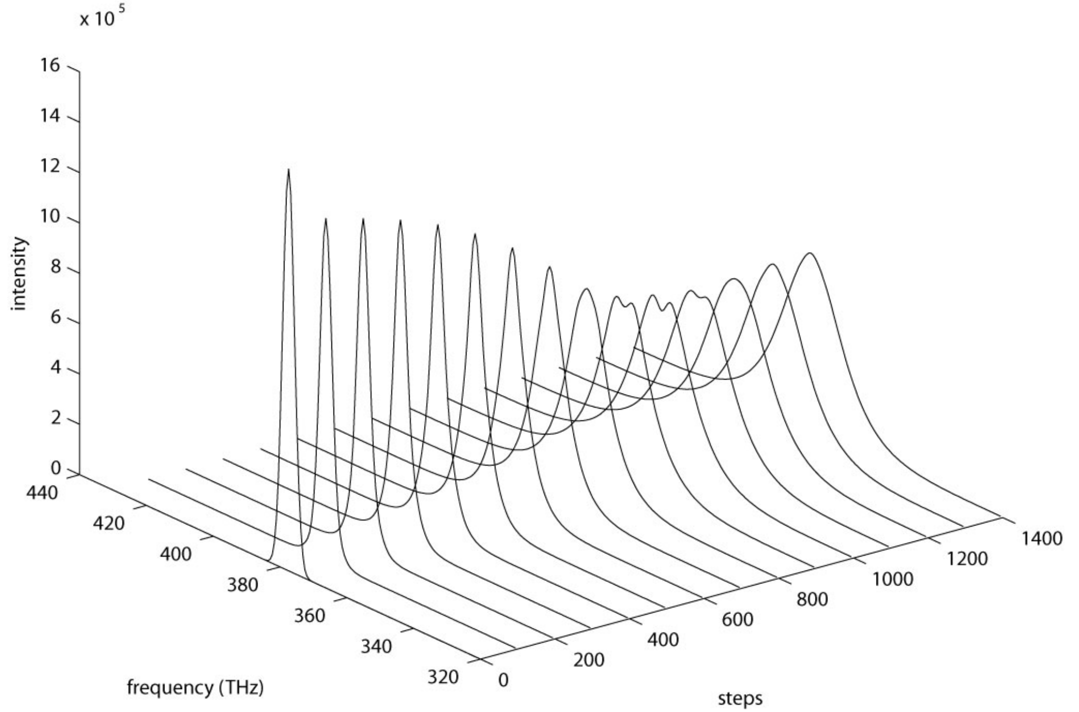



frequency

# Gain, SAM, GVD, and SPM




pulse width




frequency

# What is minimized?

- Cavity loss?
- Gain medium population inversion?
- Pulse width?

# Haus' Master Equation

master equation with GVD and SPM:

$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \Big(\frac{1}{\Omega_f^2} + \underbrace{jD}_{\text{GVD}}\Big)\frac{\delta^2}{\delta t^2}a + (\gamma \underbrace{-j\delta)|a|}_{\text{SPM}}{}^2 a$$

Gain depletion

$$g = g(T) = \frac{g_0}{1 + \frac{E_P(T)}{E_{sat}}}$$

where

$$E_P(T) \;=\; \int_{-\infty}^{+\infty} |A(T,t)|^2 dt$$

$$E_P(T) \;=\; \Delta t \sum_{n=0}^{N} |A(T,n)|^2$$

# Complex Ginzburg-Landau Equation

master equation with GVD and SPM:

$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \Big(\underbrace{\frac{1}{\Omega_f^2} + jD}_{\text{GVD}}\Big)\frac{\delta^2}{\delta t^2}a + (\gamma \underbrace{- j\delta}_{\text{SPM}})|a|^2 a$$

Gain depletion

$$g = g(T) = \frac{g_0}{1 + \frac{E_P(T)}{E_{sat}}}$$

where

$$\begin{aligned} E_P(T) &= \int_{-\infty}^{+\infty} |A(T,t)|^2 dt \\ E_P(T) &= \Delta t \sum_{n=0}^{N} |A(T,n)|^2 \end{aligned}$$

# Complex Ginzburg-Landau Equation

master equation:

$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \left(\frac{1}{\Omega_f^2} + jD\right)\frac{\delta^2}{\delta t^2}a + (\gamma - j\delta)|a|^2 a$$

soliton like pulse

$$a(t) = A_0\ \mathrm{sech}^{(1+j\beta)}\left(\frac{t}{\tau}\right)$$

CW solution

$$a(t) = A_0 \exp(-j\omega t)$$

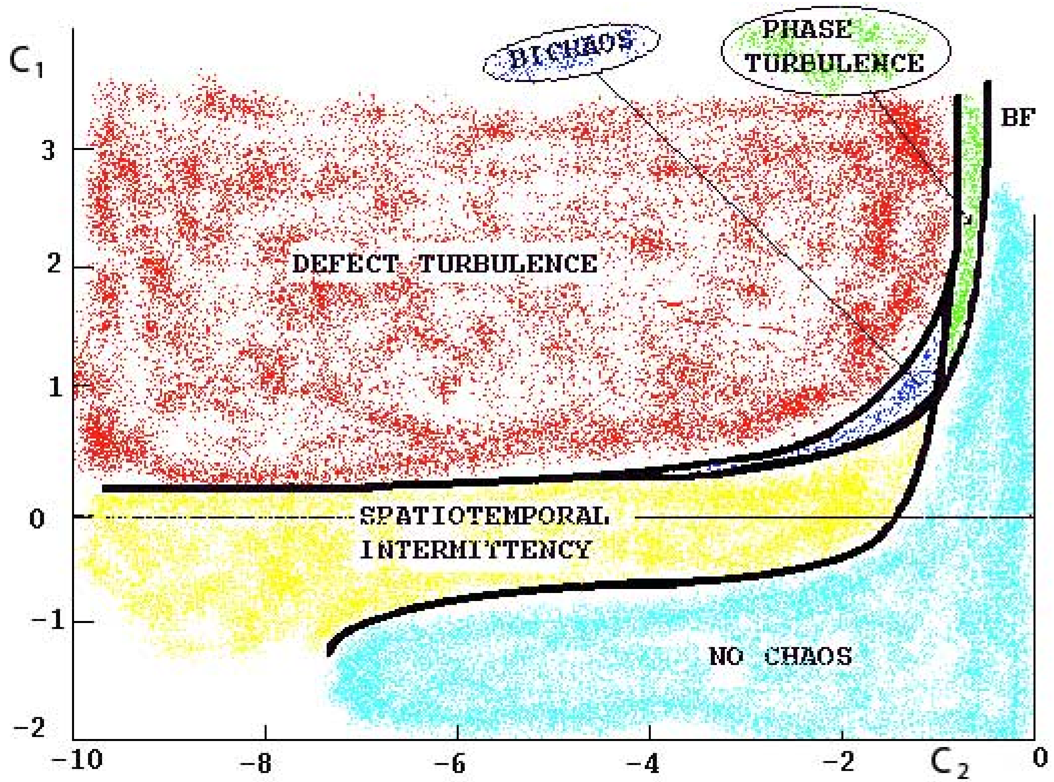
general CGLE

$$\frac{\partial}{\partial T}A = A + (1 + jc_1)\frac{\partial^2}{\partial t^2}A - (1 + jc_2)|A|^2 A$$

# Complex Ginzburg-Landau Equation

general CGLE

$$\frac{\partial}{\partial T}A = A + (1 + jc_1)\frac{\partial^2}{\partial t^2}A - (1 + jc_2)|A|^2 A$$



# Lyapunov Function

soliton like pulse

$$a(t) = A_0 \ \mathsf{sech}^{(1+j\beta)}\left(\frac{t}{\tau}\right)$$

CW solution

$$a(t) = A_0 \, \mathsf{exp}(-j\omega t)$$

A good approximate Lyapunov function is known for a CW stationary solution.

No Lyapunov function is known for soliton solutions—too close to chaos. IMHO

# Simplify Equation

$$\frac{1}{T_R}\frac{\delta}{\delta T}a = (g-l)a + \left(\frac{1}{\Omega_f^2} + \underline{jD}\right)\underline{\frac{\delta^2}{\delta t^2}}a + (\gamma - \underline{j\delta)|a|^2}a$$

$$\frac{\partial}{\partial T}A(T,t) = jD\frac{\partial^2}{\partial t^2}A - j\delta|A|^2 A$$

ignore gain depletion, BW filtering and SAM

# Non-linear Schrodinger Equation

$$\frac{\partial}{\partial T}A(T,t) = jD\frac{\partial^2}{\partial t^2}A - j\delta|A|^2 A$$

Lyapunov Function:

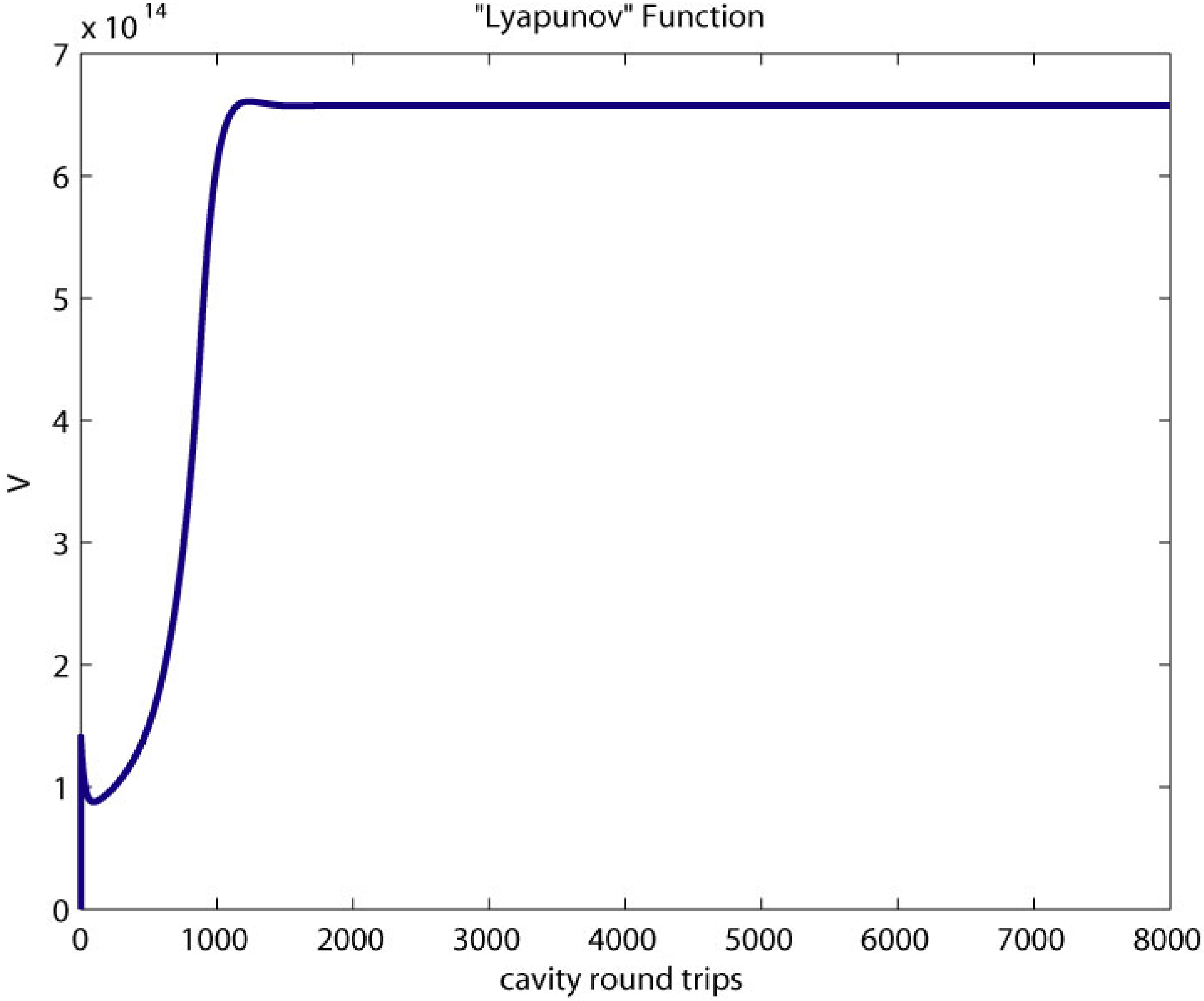
$$V = \int_{-\infty}^{+\infty} dt \Big[-D|A|^2 + \frac{\delta}{4}|A|^4 a + |\frac{\partial}{\partial t}A|^2\Big]$$

Looks a lot like minimizing the action.

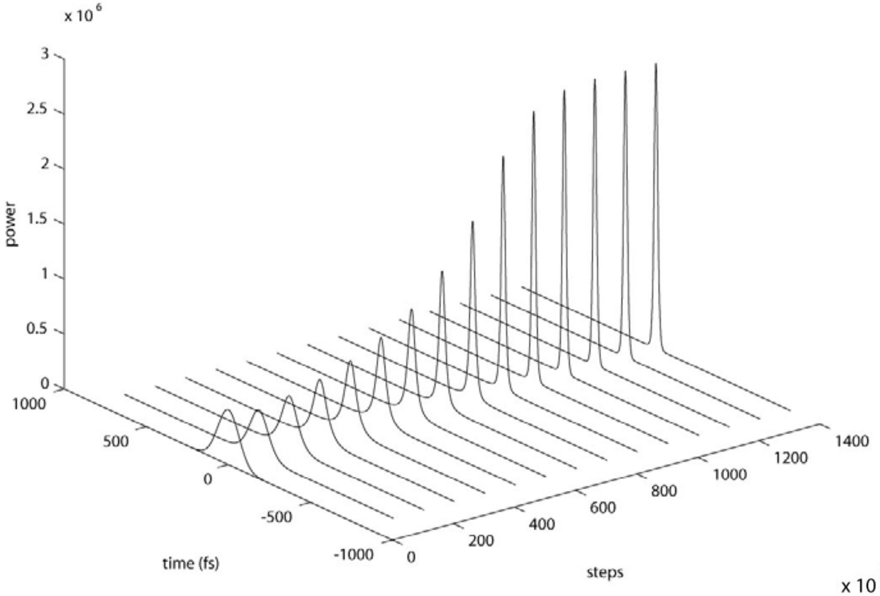
# Numerical Confirmation

Try Lyapunov function in simulator.

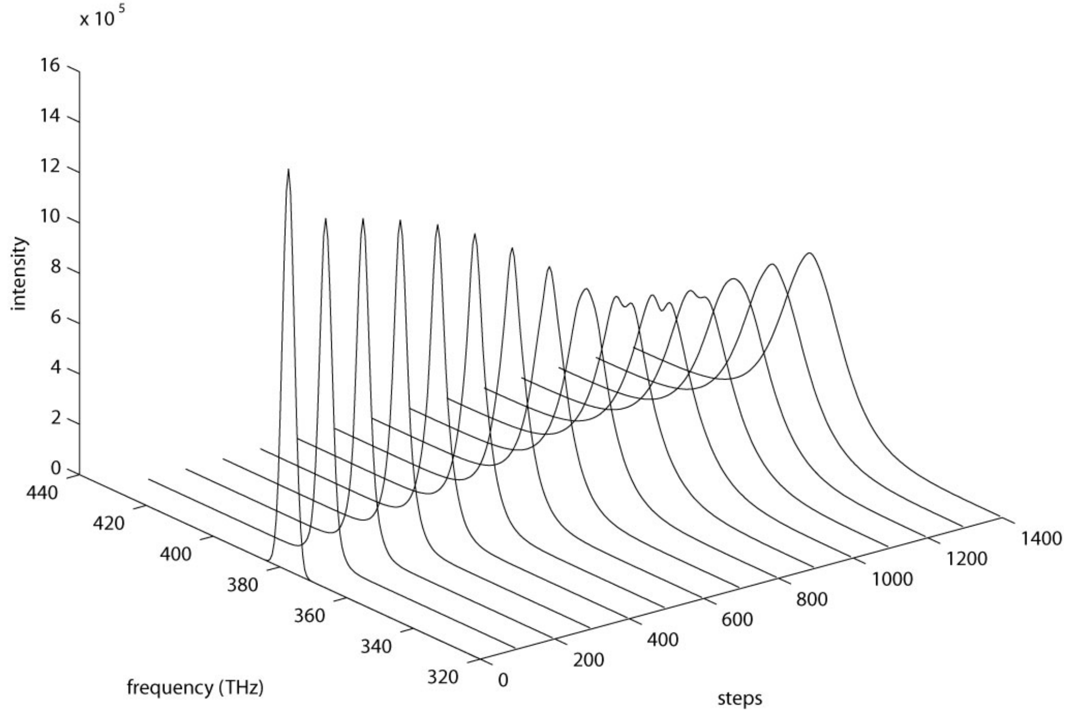
# NLSE Lyapunov function on CGLE with gain saturation



# Gain, SAM, GVD, and SPM

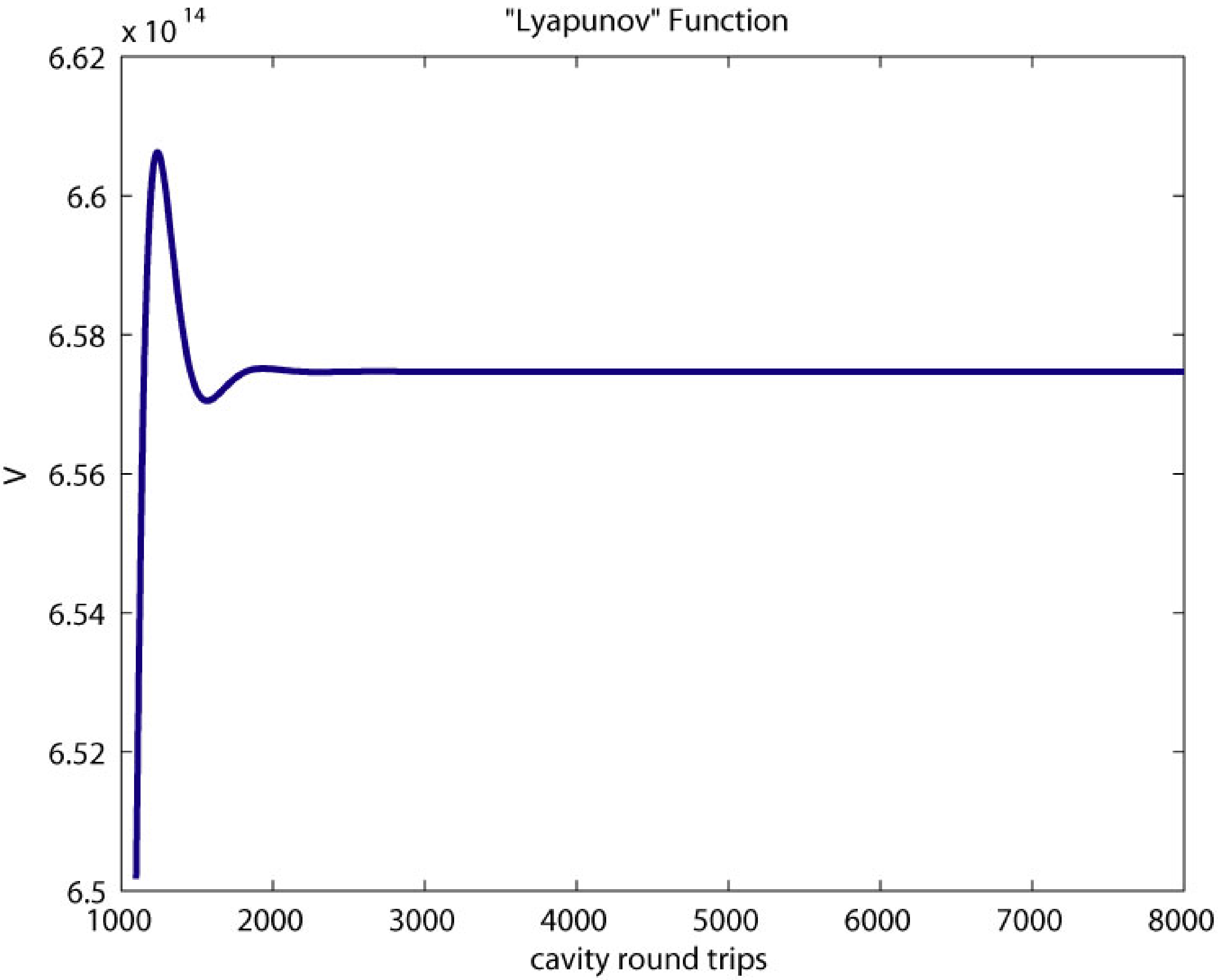



pulse width

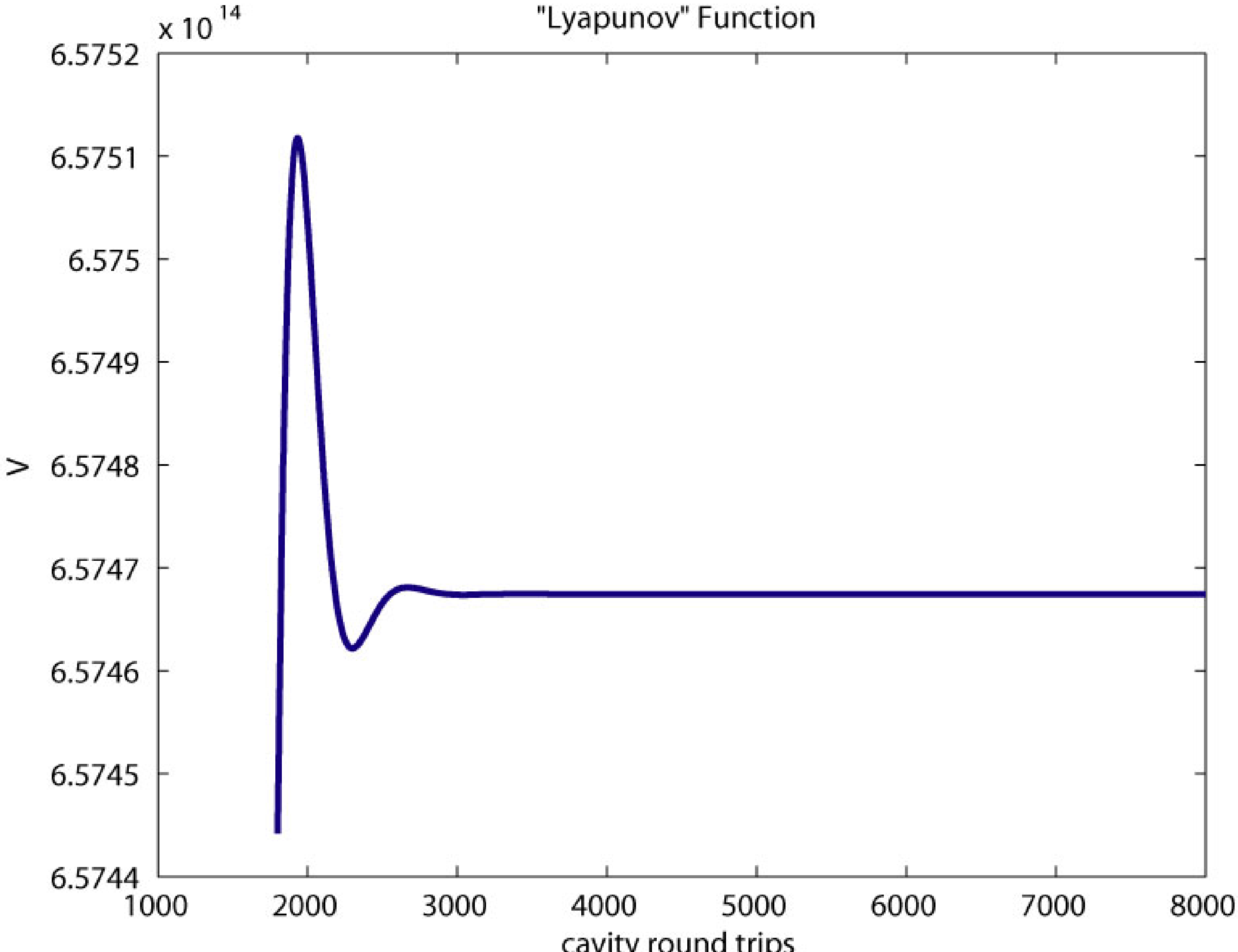



frequency

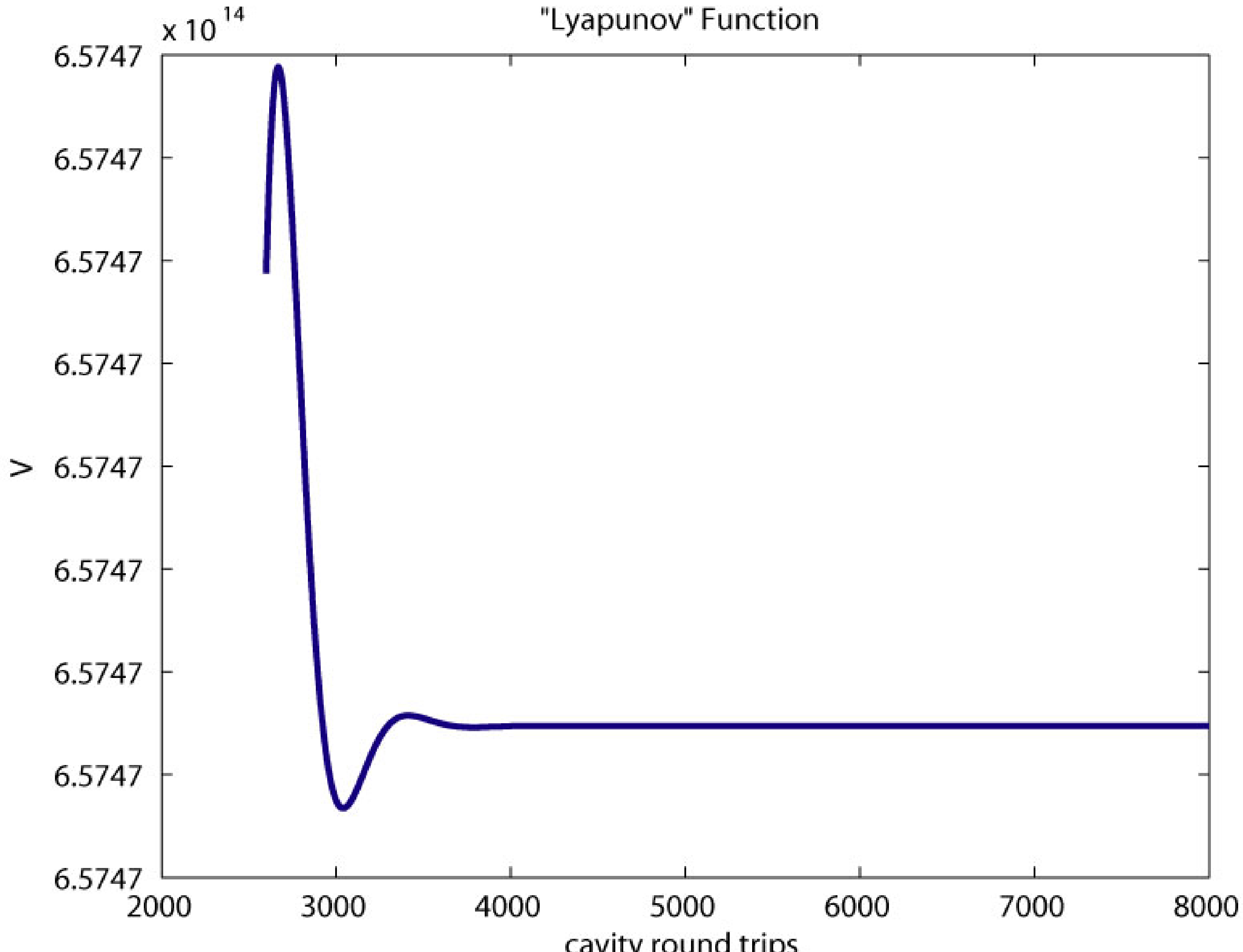
# Zoom in on Hump



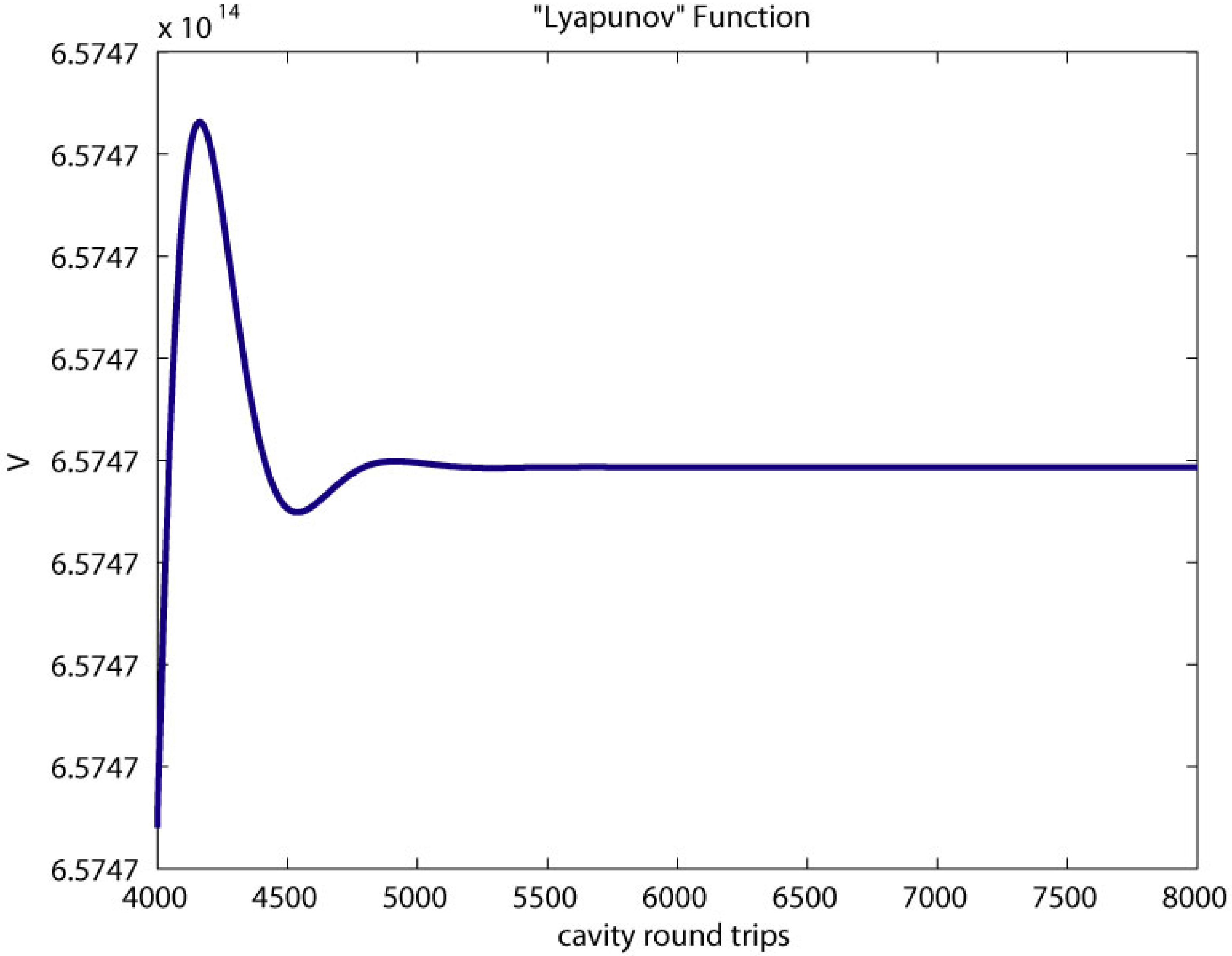
# Zoom in on 2nd Hump



# Zoom in on 3rd Hump



# Zoom in on 4$^{th}$ Hump



# Conclusions

- We found a fractal.
- The NLSE approximate Lyapunov function isn’t valid far away from the soliton solution.
- Is a numerical stability analysis of the CGLE sufficient?