MAS - 864

8.1

$$y(x+h) = y(x) + \frac{h}{2}\left\{ f(x, y(x)) + f\left[x+h, y(x) + hf(x, y(x))\right] \right\}$$

Now $f\left[x+h, y(x) + hf(x, y(x))\right]$

$$= f(x, y(x)) + h \frac{d}{dh} f\left[x+h, y(x) + hf(x, y(x))\right]$$

$$= f(x, y(x)) + h\left[\frac{\partial f}{\partial x} 1 + \frac{\partial f}{\partial y}\left(f(x, y(x))\right)\right]$$

Hence $y(x+h) = y(x) + \frac{h}{2}\left\{ f(x, y(x)) + f(x, y(x)) + h\left[\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} f(x, y(x))\right]\right\}$

$$= y(x) + \frac{h}{2}\left[f(x, y(x))\left[2 + h\frac{\partial f}{\partial x}\right] + h\frac{\partial f}{\partial y}\right] + O(h^3)$$

8.2 $\ddot{y} + y = 0$, $y(0) = 1$

$\dot{y}(0) = 0$

If $\frac{d^2y}{dt^2} = -y$, then $y = A\sin(t+\phi)$, $A\sin(\phi) = 1$

$\dot{y} = A\cos(t+\phi)$, $A\cos(\phi) = 0$

$\Rightarrow \phi = \pi/4$, $A = 1$

Hence $y = \cos(t)$

Using Euler method, $y(x+h) = y(x) + hf(x, y(x))$

$\Rightarrow \cos(t+h) = \cos(t) - \cancel{h\sin(t)}$?

Use $\alpha = \dot{y}$

$\dot{\alpha} = \ddot{y}$