9.1) $\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} + \gamma \frac{\partial}{\partial t} \frac{\partial^2 u}{\partial x^2}$ Damped wave. Solution domain on interval $[0, 1]$

a) Use Galerkin method to find approximate system of DE's

$$\int R(\vec{x}) w_i(\vec{x}) d\vec{x} = 0$$

where, $w_i(\vec{x}) = \frac{\partial u}{\partial a_i} = \varphi_i(\vec{x})$

$$R(\vec{x}) = \frac{\partial^2 u}{\partial t^2} - v^2 \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial}{\partial t} \frac{\partial^2 u}{\partial x^2} - f(\vec{x}, \epsilon)$$ → 0 no forcing term

$$\Rightarrow \int \left(\frac{\partial^2 u}{\partial t^2} - v^2 \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial}{\partial t} \frac{\partial^2 u}{\partial x^2} \right) \varphi_j(\vec{x}) dx = 0$$

let $u(x,t) = \sum_i a_i(t) \varphi_i(x)$ ← basis functions satisfy boundary conditions, complete set

$$\frac{\partial u}{\partial t} = \sum_i \frac{da_i}{dt} \varphi_i \qquad \frac{\partial u}{\partial x} = \sum_i a_i \frac{d\varphi_i}{dx}$$

$$\frac{\partial^2 u}{\partial t^2} = \sum_i \frac{d^2 a_i}{dt^2} \varphi_i \qquad \frac{\partial^2 u}{\partial x^2} = \sum_i a_i \frac{d^2 \varphi_i}{dx^2}$$

$$\frac{\partial}{\partial t} \frac{\partial^2 u}{\partial x^2} = \sum \frac{da_i}{dt} \frac{d^2 \varphi_i}{dx^2}$$

$$\Rightarrow \sum_i \int \left(\frac{d^2 a_i}{dt^2} \varphi_i \varphi_j - v^2 a_i \frac{d^2 \varphi_i}{dx^2} \varphi_j - \gamma \frac{da_i}{dt} \frac{d^2 \varphi_i}{dx^2} \varphi_j \right) dx = 0$$

$$\Rightarrow \sum_i \int \left[\left(\frac{d^2 a_i}{dt^2} \varphi_i \varphi_j - \gamma \frac{da_i}{dt} \frac{d^2 \varphi_i}{dx^2} \varphi_j \right) - v^2 a_i \frac{d^2 \varphi_i}{dx^2} \varphi_j \right] dx = 0$$

$$\vec{A} = \int \varphi_i \varphi_j \, dx$$

$$\vec{B} = \int \gamma \frac{d^2 \varphi_i}{dx^2} \varphi_j \, dx$$

$$\vec{C} = \int v^2 \frac{d^2 \varphi_i}{dx^2} \varphi_j \, dx$$

$$\Rightarrow \vec{A} \cdot \frac{d^2 a_i}{dt^2} - \vec{B} \cdot \frac{da_i}{dt} - \vec{C} a_i = 0$$

Q: on pg 113 I see integration by parts is used to reduce spatial order to 1 for 2. Not sure if I should separate this out.