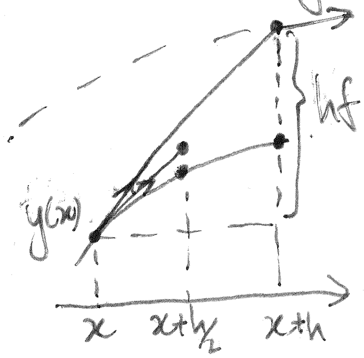
Meng Yee CHUAH MAS. 864 11 March 2014

(7.1) $y(x+h) = y(x) + \frac{h}{2}\{f(x,y(x)) + f[x+h, y(x) + hf(x,y(x))]\}$



(7.10)

Taylor expansion as a function of h,

$f[x+h, y(x) + hf(x,y(x))]$

$$= f(x,y(x)) + h\frac{d}{dh}f[x+h, y(x)+hf(x,y(x))]_{h=0} + O(h^2)$$

$$= f(x,y(x)) + h\left[\frac{\delta f}{\delta x} + f(x,y(x))\frac{\delta f}{\delta y}\right] + O(h^2)$$

$$\Rightarrow y(x+h) = y(x) + \frac{h}{2}\left\{f(x,y(x)) + f(x,y(x)) + h\left[\frac{\delta f}{\delta x} + f(x,y(x))\frac{\delta f}{\delta y} + O(h^2)\right]\right\}$$

$$= y(x) + hf(x,y(x)) + \frac{h^2}{2}\left[\frac{\delta f}{\delta x} + f(x,y(x))\frac{\delta f}{\delta y}\right] + O(h^3)$$

There is no second-order approximation error of the Heun method.

(7.2) $\ddot{y} + y = 0$

Let $y = c_1 \sin(t) + c_2 \cos(t)$

$\dot{y} = c_1 \cos(t) - c_2 \sin(t)$

$\ddot{y} = -c_1 \sin(t) - c_2 \cos(t) = -y$

$\ddot{y} + y = -c_1 \sin(t) - c_2 \cos(t) + c_1 \sin(t) + c_2 \cos(t) = 0$

For $y(0) = 1 = c_2$

For $\dot{y}(0) = 0 = c_1$

The solution is $y = \cos(t)$