C.1 a)

$$\ln\left\langle e^{ikx}\right\rangle = \sum_{0} \frac{(ik)^n}{n!} c_n$$ - series expansion for cumulants

~~power~~ series expansion

$$\exp\left(\sum_1 \frac{(ik)^n}{n!} c_n\right) = \sum_0^\infty \frac{1}{m!}\left(\sum_1 \frac{(ik)^n}{n!} c_n\right)^m$$

$$= 1 + ik\langle x\rangle - \frac{k^2}{2}\langle x^2\rangle - \frac{ik^3}{6}\langle x^3\rangle$$

def of cumulant → $\langle x\rangle = c_1$, $\langle x^2\rangle = c_2 + c_1^2$, $\langle x^3\rangle = c_3 + 3c_2c_1 + c_1^3$

$$c_1 = \langle x\rangle$$

$$c_2 = \langle x^2\rangle - c_1^2 = \langle x^2\rangle - \langle x\rangle^2$$

$$c_3 = \langle x^3\rangle - 3c_2c_1 - c_1^3 = \langle x^3\rangle - 3\langle x\rangle\left(\langle x^2\rangle - \langle x\rangle^2\right) - \langle x\rangle^3$$

$$= \langle x^3\rangle - 3\langle x\rangle\langle x^2\rangle + 2\langle x\rangle^3$$

b)

$$\left\langle e^{ikx}\right\rangle$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} \exp\left(-\frac{(x-x_0)^2}{2\sigma^2} + ikx\right) x_n \, dx$$

$$= \frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{\infty} e^{\frac{-(x-x_0)^2}{2\sigma^2}} x^n \, dx$$

for $n = 1$; $\langle e^{ikx}\rangle = x_0 = c_1$

" 2; " $= x_0^2 + \sigma^2 = c_2 + c_1^2$

$$c_2 = \sigma^2$$

" $3+$; " $= x_0^3 + 3x_0\sigma^2 = c_3 + 3c_2c_1 + c_1^3$

$$c_3 = 0$$ and higher

6.2) a) det is parallel area so coordinate transform function of jacobian det

6.2) b) $p(y_1, y_2) = |J| \, p(x_1, x_2)$

$$J = \begin{bmatrix} \frac{\delta y_1}{\delta x_1} & \frac{\delta y_1}{\delta x_2} \\ \frac{\delta y_2}{\delta x_1} & \frac{\delta y_2}{\delta x_2} \end{bmatrix} = \begin{bmatrix} \frac{-\sin x_2}{x_1 \sqrt{-2 \ln x_1}} & \cos x_2 \sqrt{-2 \ln x_1} \\ \frac{-\cos x_2}{x_1 \sqrt{-2 \ln x_1}} & -\sin x_1 \sqrt{-2 \ln x_1} \end{bmatrix}$$

$$J_{det} = \left| \frac{\cos^2 x_2}{x_1} + \frac{\sin^2 x_2}{x_1} \right| = \left| \frac{1}{x_1} \right|$$

$$p(y_1, y_2) = \left| \frac{1}{x_1} \right| p_1(x_1, x_2)$$

c)

6.3 a) Feed back Polynomial x^4+x^3+1 ; period 15

$2^4-1=15$

check $x^{15}+1$

```
0010        [scribbled]   0001
0100        [scribbled]   0010
1001        ?100          0100
0011        1001            3
0110        0011
1101        0110            2
1010        1101
0101        1010            1
1011        0101
0111        1011
1111        0111
1110        1111
1100        1110
1000        1100
0001        1000
0010        ?001
```

00	→ 0
11	= 0
01	= 1
10	= 1

b)

$$(f)\left(\text{seconds in } 10^{10} \ldots\right) = 3.154\times10^{26}$$

$$\log_2(3.144\times10^{26}) = \text{register of } 88$$

```
     0001           0001
     0010           0011
     0100           0111
     1001           1111
     0011           1110
     0110           1101
15   1101      15   1010
     1010           0101
     0101           1011
     1011           0110
     0111           1100
     1111           1001
     1110           0010
     1100           0100
     1000           1000
     0001           0001
```