

(6.1)

$$f(x+h, y(x) + hf(x, y(x))) = f(x, y) + h \left. \frac{d}{dh} f(x+h, y+hf(x, y)) \right|_{h=0} + O(h^2)$$
$$= f(x, y) + h \left(\frac{\partial f}{\partial x} + f \frac{\partial f}{\partial y} \right) = O(h^2)$$

ins into original eqn

$$y(x+h) = y(x) + hf(x, y) + \frac{h^2}{2} \left(\frac{\partial f}{\partial x} + f \frac{\partial f}{\partial y} \right) + O(h^3)$$

2nd order

7.2

euler ~~pseudo~~code (stepsnum)

~~int steps =~~ ; int totalSteps = stepsnum

float f0 = 1 ; y = cos()

float array f[i] = f0

for (i, ~~array~~ i < totalSteps, i++)

~~float array f[i] = f0~~

f[i+1] = f[i] + h * y(f[i])

print f[i+1]

RK4 pseudo code (stepsnum)

int totalSteps = (stepsnum)

float array [3] k ; float f0 = 1 ; y = cos()

float array f[i] = f0

for (i, i < totalSteps, i++)

k[0] = h * y(f[i])

k[1] = h * y(f[i] + k[0]/5)

k[2] = h * y(f[i] + k[1]/5)

k[3] = h * y(f[i] + k[2])

f[i+1] = f[i] + k[0]/6 + k[1]/3 + k[2]/3 + k[3]/6

print f[i+1];