9.1) a)

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} + \gamma \frac{\partial}{\partial t}\frac{\partial^2 u}{\partial x^2} \quad \text{over } [0,1]$$

~~basis functions~~

Gahlerkin in 1D

$$0 = \int_0^1 \Big(R(x,t) \Big) \varphi_j(x)\, dx$$

$$= \int_0^1 \left(v^2 \frac{\partial^2 u}{\partial x^2} + \gamma \frac{\partial}{\partial t}\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial t^2} \right) \varphi_j \, dx \; ;$$

plug in $\quad u(x,t) = \sum_i a_i(t)\, \varphi_i(x)$;

$$0 = \sum_i \int_0^1 \left(v^2 a_i(t)\, \varphi_j \frac{d^2 \varphi_i(x)}{dx^2} + \gamma \varphi_j \frac{d a_i(t)}{dt} \frac{d^2 \varphi_i(x)}{dx^2} - \varphi_i(x) \varphi_j \frac{d^2 a_i(t)}{dt^2} \right) dx$$

in Matrix form;

$$A_{ij} = \int_0^1 \varphi_i(x)\varphi_j(x)\, dx$$

$$B_{ij} = \int_0^1 \varphi_j \frac{d^2 \varphi_i(x)}{dx^2}$$

$$C_{ij} = \int_0^1 \varphi_j \frac{d^2 \varphi_i(x)}{dx^2} = B_{ij}$$

$$\boxed{\text{matrix form:} \quad -A_{ij} \frac{d^2 \vec{a}}{dt^2} + \gamma B_{ij} \frac{d\vec{a}}{dt^2} + v^2 B_{ij} \vec{a}}$$

b) if we take

$$\varphi_i = \begin{cases} \dfrac{x - x_{i-h}}{x_i - x_{i-h}} & x_{i-h} \le x \le x_i \\[2ex] \dfrac{x_{i+h} - x}{x_{i+h} - x_i} & x_i \le x \le x_i + h \end{cases}$$

so

$$A_{i,i} = \int_{x_i - h}^{x_i + h} \varphi_i \varphi_i \, dx$$

$$= \int_{x_i - h}^{x_i} \left(\frac{x - x_i - h}{h}\right)^2 dx + \int_{x_i}^{x_i + h} \left(\frac{-x + x_i + h}{h}\right)^2 dx$$

$$B_{i,j} = \int_{x_i - h}^{x_i} \left(\frac{1}{h}\right)^2 dx + \int_{x_i}^{x_i + h} \left(\frac{1}{h}\right)^2 dx$$