12.1) unitary condition: $M^{+}M = I$ where $M^{+}_{ij} = M^{*}_{ji}$
adjoint

$$M^{+}M = A_{fk} = \sum_{n} M^{+}_{fn} M_{kn}$$
$$= \sum_{n} M^{*}_{nf} M_{kn}$$

DFT for g_k is

$$g_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{\left(\frac{2\pi i}{N} fk\right)} g_k \;;$$

so

$$A_{fk} = \sum_{n=0}^{N-1} \frac{1}{N} e^{\frac{2\pi}{N} i n(k-f)}$$

at $k=f$; $A_{fk} = \sum_{n=0}^{N-1} \frac{1}{N} = 1$

at $k \neq f$, exponencial sums to 0 due to periodic factor

so, $M^{+}M = I$

12.3) a)

$\sigma^2 = \langle x_1^2 \rangle = 1$

$\langle x_2^2 \rangle = 1$

$\langle x_1 \rangle \langle x_2 \rangle = 0$

$x_3 = \langle (x_1 + x_2)^2 \rangle = \cancel{\langle x_1^2 \rangle}^{1} + \cancel{\langle x_2^2 \rangle}^{1} = 2$

$$C(\vec{x}) = \begin{pmatrix} \sigma(x_1, x_1) & \sigma(x_1, x_2) & \sigma(x_1, x_3) \\ \sigma(x_2, x_1) & \sigma(x_2, x_2) & \sigma(x_2, x_3) \\ \sigma(x_3, x_1) & \sigma(x_3, x_2) & \sigma(x_3, x_3) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 1+0 \\ 0 & 1 & 1+0 \\ 1+0 & 1+0 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

b)

$$0 = \det \begin{pmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 1 \\ 1 & 1 & 2-\lambda \end{pmatrix} =$$

$(1-\lambda)\big((1-\lambda)(2-\lambda) - 1\big) + 0 + \big(-(1-\lambda)\big)$; ...

$\lambda = 1, 0, 3$