Nature of Mathematical Methods - Transforms Ch. 12

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(12.) Prove DFT is unitary:

If DFT is unitary, then the transpose times the original matrix is the identity matrix: $M^T M = I$.

Suppose $M = \frac{1}{\sqrt{N}} e^{2\pi i k n / N}$

Then $M^T M = \sum_{j=0}^{N-1} M_{kj}^{+} M_{jn}$

$$= \frac{1}{N} \sum e^{2\pi i k n/N} e^{-2\pi i k n/N}$$

$$= \frac{1}{N} \sum e^{2\pi i (n-k)}$$

$\Rightarrow$ only elements on diagonal $\neq 0$.

$(n \neq k = 0)$ ✓

(This from class:) off-diagonal is geometric sum:

$$\frac{1-x^N}{1-x} = 1 + x + x^2 \dots + x^{N-1} \Rightarrow \frac{1}{N} \frac{\left[1 - e^{2\pi i (k-n)}\right]}{e^{2\pi i (k-N)/N}}$$

$$= \frac{1}{N} \frac{1 - \cos(2\pi(k-n)) - i \sin(2\pi(k-n))}{e^{2\pi i (k-n)/N}}$$

$$= 0$$ ✓

(12.2) Calculate the inverse wavelet transform using Daubechie's 4^{th} order coefficients of a vector length 2^{12} with 1 in the 5^{th} & 30^{th} places & 0 elsewhere

<u>Daubechie's wavelets:</u>

$$y_n = \sum_{i=0}^{M-1} b_i x_{n-i}$$

<u>D4 scaling function coefficients:</u>

$$h_0 = \frac{1+\sqrt{3}}{4\sqrt{2}}$$

$$h_1 = \frac{3+\sqrt{3}}{4\sqrt{2}}$$

$$h_2 = \frac{3-\sqrt{3}}{4\sqrt{2}}$$

$$h_3 = \frac{1-\sqrt{3}}{4\sqrt{2}}$$

<u>Wavelet function coefficients:</u>

$$g_0 = h_3$$

$$g_1 = -h_2$$

$$g_2 = h_1$$

$$g_3 = -h_0$$