NoMM Architectures - Chapter 14

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Christian Wertz

$\swarrow \left[\frac{M}{N}\right]$ notation

(14.1) "Find First 5 diagonal Padé Approximants to e^x $\left[\frac{1}{1}\right], \ldots \left[\frac{5}{5}\right]$

Evaluate at $x=1$."

Recall $y(x) = \dfrac{\sum_{n=1}^{N} a_n x^n}{1 + \sum_{m=1}^{M} b_m x^m}$ is classic Padé Approximant

$$= \underbrace{\sum_{l=0}^{L} c_l x^l}_{\text{Taylor/Maclaurin series}} = \underbrace{\sum_{l=0}^{L} \frac{x^l}{l!}}_{\text{for } e^x}$$

We find coefficients by:

$$\sum_{n=0}^{N} a_n x^n = \sum_{l=0}^{L} \frac{x^l}{l!} + \sum_{m=1}^{M} \sum_{l=0}^{L} b_m \frac{x^{l+m}}{l!}$$

Via Mathematica:

M/N	Padé	@ $x=1$	[Error]:
1/1	$\frac{2+x}{2-x}$	3	0.2817
2/2	$\frac{12+6x+x^2}{12-6x+x^2}$	2.7143	4×10^{-3}
3/3	$\frac{120+60x+12x^2+x^3}{120-60x+12x^2-x^3}$	2.7133	2.3031×10^{-5}
4/4	$\frac{x^4+20x^3+180x^2+840+1680}{x^4-20x^3+180x^2-840+1680}$	2.7183	1.1018×10^{-7}
5/5	$\frac{x^5+30x^4+420x^3+3360x^2+15120+30240}{-x^5+30x^4-420x^3+3360x^2-15120+30240}$	2.7133	2.7665×10^{-10}

14.2 (a)

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^m \\ 1 & & & & \\ \vdots & & & & \\ 1 & x_n & x_n^2 & \cdots & x_n^m \end{bmatrix}}_{\text{Vandermonde matrix}} \begin{bmatrix} a_0 \\ \vdots \\ \vdots \\ a_M \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

$M < n$

$n = 21$

$M = 5, 10, 15$

Because we are working with a non-square Vandermonde matrix we must use the pseudo-inverse, $(X^T X)^{-1} X^T$, then proceed with least squares fitting:

$$y = (X^T X)^{-1} X^T a$$