Design and Implementation of Simple Field-Oriented Control for Permanent Magnet Stepper Motors Without DQ Transformation

Wonhee Kim[1], Chuan Yang[1], and Chung Choo Chung[2]

[1]Department of Electrical Engineering, Hanyang University, Seoul 133-791 Korea
[2]Division of Electrical and Biomedical Engineering, Hanyang University, Seoul 133-791 Korea

In this paper, a simple field-oriented control (FOC) without direct quadrature (DQ) transformation is proposed for position tracking of permanent magnet stepper motors (PMSMs). Conventional FOC methods require DQ transformation to linearize the mechanical dynamics for PMSMs. In this paper, a proportional-integral-derivative controller with velocity feedforward is developed to obtain the torque modulation required to track the desired position. In addition, a new commutation scheme is proposed to generate the desired currents with the torque modulation; this commutation scheme is equivalent to the microstepping where the desired currents have time-varying amplitudes with $\pi/2$ electrical phase advance. The proposed controller method is in the form of an FOC even though DQ transformation is not used. The proposed commutation scheme likens the PMSM to a two-phase permanent magnet synchronous motor. Experimental results validate the effectiveness of the proposed method.

***Index Terms*—Field-oriented control (FOC), permanent magnet stepper motor (PMSM), proportional-integral-derivative (PID) controller.**

I. Introduction

VARIOUS field-oriented control (FOC) methods have been developed for permanent magnet stepper motors (PMSMs) [1]–[3]. The FOC maintains a zero direct current in order to maximize the torque. Typically, proportional-integral (PI) controllers are used for FOCs [1]. An optimal control with an FOC was proposed to control PMSMs with voltage and current constraints [2]. Model-based damping algorithms for both open-loop control and servo control were designed to eliminate low-speed resonance and vibration [3]. Each of these methods [1]–[3] requires direct quadrature (DQ) transformation to linearize the mechanical dynamics [4]. Using DQ transformation results in an increase of the computation required for the implementation of the control method. Moreover, it has also been reported that a closed-loop commutation delay may cause unusual qualitative changes in the behavior of PMSMs [5].

In this paper, a simple FOC without DQ transformation is proposed for the position tracking of PMSMs. A conventional proportional-integral-derivative (PID) controller is used for obtaining torque modulation for position tracking. Velocity feedforward is used in order to improve the position-tracking performance. The PID controller with velocity feedforward generates the desired torque, i.e., torque modulation. The stability of the position-tracking error dynamics with the proposed method is proven. This paper proposes a new commutation scheme to generate the desired currents for the desired torque; this commutation scheme is equivalent to microstepping where the desired currents have time-varying amplitudes with $\pi/2$ electrical phase advance. The proposed method is in the form of an FOC even though DQ transformation is not used. Thus, the proposed method can reduce the necessary computation required of the processor used to implement the control scheme so that the proposed method can be implemented by using a low-cost processor, for example, ATmega128. For low-cost and simple implementation, pulsewidth-modulation (PWM) drivers, which include PI current feedback, have been used in industry applications [6], [7]. The proposed method was validated with a commercial PWM driver having an embedded PI current feedback loop. Experimental results validate the effectiveness of the proposed method.

We summarize the primary contributions of this paper as follows.

1) The advantage of the proposed method is the achievement of an FOC without DQ transformation.
2) The proposed method is equivalent to microstepping with torque modulation.
3) The proposed commutation scheme likens a PMSM to a two-phase permanent magnet synchronous motor.

This paper is organized as follows. Section II presents the development of the proposed method, Section III presents the experimental results, and Section IV provides our conclusions.

II. Controller Design

The dynamics of a PMSM can be represented in state space such that [9]

$$\begin{aligned}\dot{\theta} &= \omega\\ \dot{\omega} &= \frac{1}{J}\left[-K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta) - B\omega - \tau_l\right]\\ \dot{i}_a &= \frac{1}{L}\left[v_a - Ri_a + K_m\omega\sin(N_r\theta)\right]\\ \dot{i}_b &= \frac{1}{L}\left[v_b - Ri_b - K_m\omega\cos(N_r\theta)\right]\end{aligned} \tag{1}$$

where v_a, v_b and i_a, i_b are the voltages and currents in phases A and B, respectively. ω is the rotor (angular) velocity, θ is the rotor (angular) position, B is the viscous friction coefficient, J is the inertia of the motor, K_m is the motor torque constant, R is the phase winding resistance, L is the phase winding inductance, and N_r is the number of rotor teeth. The load torque perturbation, denoted by τ_l, is, for analysis purposes, assumed to be zero. Since detent torque in a PMSM does not significantly affect the torque produced by the motor, it can therefore be ignored [8]. In

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addition, the magnetic coupling between the phases is also ignored, as well as the variation in inductance due to magnetic saturation. Furthermore, an ideal sinusoidal flux distribution is assumed. In a PMSM, since the electrical dynamics is much faster than the mechanical dynamics, it allows us to only consider the mechanical dynamics [10]. In industrial applications, the electrical dynamics is ignored using PWM drivers assembled with PI current feedback, i.e., a current controlled voltage source inverter [6], [7]. Therefore, in this paper, we consider the mechanical dynamics with the use of PWM drivers assembled with PI current feedback. By ignoring the electrical dynamics, the mechanical dynamics of a PMSM, represented in the state-space form, are [10], [11]

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{J}\left[-K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta) - B\omega\right].\end{aligned} \quad (2)$$

Note that the currents i_a and i_b are inputs of (2).

In a PMSM (2), the torque τ can be considered as the PMSM input such that

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{J}[\tau - B\omega]\end{aligned} \quad (3)$$

where $\tau = -K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta)$. In order to track the desired position θ_d, the position PID controller is designed such that

$$\tau = k_P(\theta_d - \theta) + k_I \int_0^t (\theta_d - \theta)dt + k_D(\dot{\theta}_d - \dot{\theta}). \quad (4)$$

Since τ is considered as an input to the system (3), the system is linear. Thus, the PID controller gains can be easily tuned for position control of the PMSM [12]. The use of only a PID controller results in a position-tracking error during a constant velocity period. To improve the position tracking, velocity feedforward is added to the PID control law (4) such that

$$\tau = k_P(\theta_d - \theta) + k_I \int_0^t (\theta_d - \theta)dt + k_D(\dot{\theta}_d - \dot{\theta}) + B\omega_d + J\dot{\omega}_d \quad (5)$$

where ω_d is the desired velocity. The tracking error $e = [e_1\ e_2\ e_3]^T$ is defined as

$$\begin{aligned}e_1 &= \int_0^t (\theta_d - \theta)dt \\ e_2 &= \theta_d - \theta \\ e_3 &= \omega_d - \omega.\end{aligned} \quad (6)$$

The dynamics of the tracking error is

$$\begin{aligned}\dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= \dot{\omega}_d - \frac{1}{J}[\tau - B\omega].\end{aligned} \quad (7)$$

Proposition 1: Suppose that the control law (5) is used in (3). If the control gains $k_P, k_I, \text{and} k_D$ are designed such that A_e is a Hurwitz matrix such that

$$A_e = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\frac{k_I}{J} & -\frac{k_P}{J} & -\frac{k_D+B}{J} \end{bmatrix} \quad (8)$$

then the origin of (7) is exponentially stable.

Proof: Substituting (5) into (7) results in

$$\begin{aligned}\dot{e}_1 &= e_2 \\ \dot{e}_2 &= e_3 \\ \dot{e}_3 &= -\frac{k_I}{J}e_1 - \frac{k_P}{J}e_2 - \frac{k_D+B}{J}e_3.\end{aligned} \quad (9)$$

Equation (9) can be rewritten as

$$\dot{e} = A_e e. \quad (10)$$

If the control gains $k_P, k_I, \text{and} k_D$ are designed such that A_e is Hurwitz, then the tracking error e exponentially converges to zero. ■

In the PMSM (2), torque τ is generated by the current by

$$\tau = -K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta).$$

The control law (5) is designed under the assumption that torque τ is the input in (3). The actual input of the PMSM (2) is, however, not torque τ but the currents i_a, i_b. In order to generate the torque (5) and use (5) as torque modulation, we propose a new commutation scheme defined by

$$i_{a_d} = -\frac{\tau}{K_m}\sin(N_r\theta), \quad i_{b_d} = \frac{\tau}{K_m}\cos(N_r\theta). \quad (11)$$

A similar result was reported under the assumption that the direct current is held at zero in order to maintain constant torque [8].

From now, the commutation scheme (11) is analyzed compared with conventional microstepping. In conventional microstepping using closed-loop PI current control [6], the desired current inputs $i_{a_{ms_d}}, i_{b_{ms_d}}$ are defined by

$$i_{a_{ms_d}} = I_{\max}\cos(N_r\theta_d), \quad i_{b_{ms_d}} = I_{\max}\sin(N_r\theta_d) \quad (12)$$

which are the inputs to the PMSM (2). From the following relationships:

$$\begin{aligned}i_{a_d} &= -\frac{\tau}{K_m}\sin(N_r\theta) = \frac{\tau}{K_m}\cos\left(N_r\theta + \frac{\pi}{2}\right) \\ &= \frac{\tau}{K_m}\cos\left(N_r\theta_{ms_d}\right) \\ i_{b_d} &= \frac{\tau}{K_m}\cos(N_r\theta) = \frac{\tau}{K_m}\sin\left(N_r\theta + \frac{\pi}{2}\right) \\ &= \frac{\tau}{K_m}\sin\left(N_r\theta_{ms_d}\right)\end{aligned} \quad (13)$$

we are able to observe that the proposed commutation scheme is equivalent to conventional microstepping where the desired currents have time-varying amplitudes with $\pi/2$ electrical phase advance. The generated desired electrical position $N_r\theta_d$ by the desired currents is illustrated in Fig. 1. Note that the desired position θ_{ms_d} in the commutation scheme is not the desired posi-

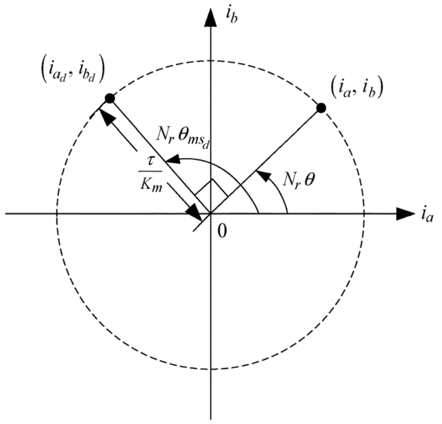

Fig. 1. Generated desired electrical position $N_r\theta_d$ by the desired phase currents (11).

tion θ_d in the control law (5). However, θ_{ms_d} is the same as θ_d in microstepping (12) if the microstepping (12) is used in the PMSM (2). The DQ transformation [4] for the currents is defined, respectively, as

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta) & \sin(N_r\theta) \\ -\sin(N_r\theta) & \cos(N_r\theta) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix}. \tag{14}$$

Applying the DQ transformation to the desired currents (11), they become

$$i_{d_d} = 0, \quad i_{q_d} = \frac{\tau}{K_m}. \tag{15}$$

Therefore, the proposed control is clearly in the form of the FOC. It turns out that the proposed commutation scheme enables the PMSM to operate in a similar manner as a two-phase permanent magnet synchronous motor.

We summarize this primary result by Proposition 2.

Proposition 2: Consider the PMSM (2). Suppose that the control law (5) and the commutation scheme (11) are used in the PMSM (2). If the control gains $k_P, k_I, \mathrm{and} k_D$ in (5) are designed such that A_e is Hurwitz, then the tracking error e exponentially converge to zero. Furthermore, zero direct current is maintained by the proposed method (5) and (11). ■

Remark 1: In conventional microstepping, the effect of back EMFs is reduced by the inner-loop PI current feedback. In industry applications, many PWM drivers with a PI current feedback loop are available as low-cost simple implementations [6], [7]. In this paper, the proposed method uses a PWM driver with an embedded PI current feedback for experiments. ■

III. Experimental Results

Experiments were executed to evaluate the performances of the proposed method. The experimental setup is shown in Fig. 2. A two-phase PMSM (PK266-01B manufactured by Oriental Motor Co., Tokyo, Japan) was used. For position feedback, an incremental optical encoder (8000 lines/rev) was used. Quadrature signals were used to obtain $\times$ 4 resolution. The sampling time was 0.35 ms, and 8-bit digital-to-analog converters were used. For the implementation of the position PID with velocity feedforward and the commutation scheme in ATmega128 (ATMEL co.), an S-function coded in C language was used. Two L292 (STMicroelectronics Co., Geneva, Switzerland) PWM motor drivers with embedded PI current feedback loops were used. The block diagram of the experimental setup is depicted in Fig. 3. The PMSM parameters and the control gain are $J = 8\times10^{-5}\ \mathrm{kg\cdot m^2}$, $K_m = 0.51\ \mathrm{N\cdot m/A}$, $N_r = 50$, $B = 8\times10^{-3}\ \mathrm{N\cdot m\cdot s/rad}$, $k_P = 50$, $k_I = 2$, $k_D = 40$, and $I_{\max} = 0.2$, respectively. Fig. 4 shows the desired position profile used.

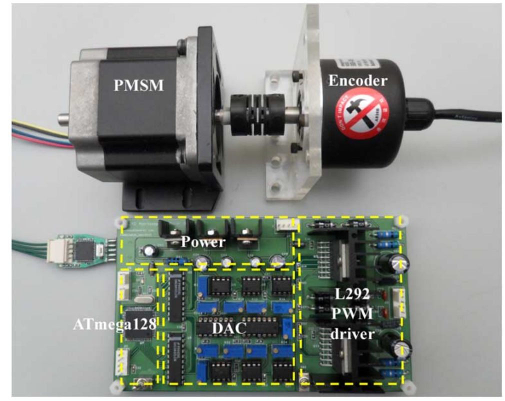

Fig. 2. Photograph of the experimental setup.

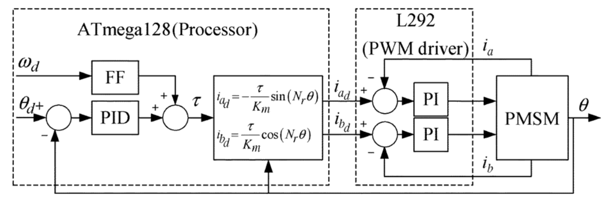

Fig. 3. Block diagram of the experimental setup.

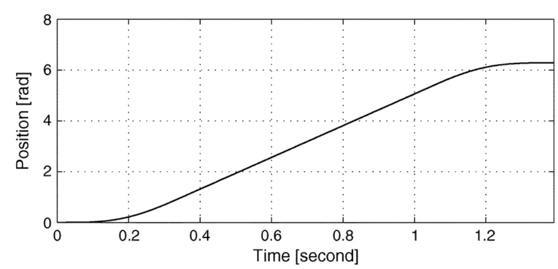

Fig. 4. Desired position profile θ_d.

Experiments were performed for the following three cases: 1) only microstepping (12) was used with PI current feedback; 2) the PID controller without the velocity feedforward (4) and the commutation scheme (11) were used with the PI current feedback; and 3) the PID controller with the velocity feedforward (5) and the commutation scheme (11) were used with the PI current feedback. The position-tracking errors of each of the three cases, 1, 2, and 3 are shown in Fig. 5(a)–(c), respectively. The unavoidable position ripples appeared due to a variety of reasons: the encoder coupling effect, the PWM driver noise, modeling uncertainty, nonideal sinusoidal flux distribution, or the cogging torque in these experiments. From Fig. 5(a) and (b), it is observed that although the PID controller improved the position-tracking performance as compared with PI current control microstepping, the position-tracking error during a constant velocity period was not rejected in case 2. The proposed method mathematically guarantees that the tracking error e exponentially converges to zero only with velocity feedforward. Therefore, although there is an improvement in reducing the steady-state position-tracking error during the constant velocity period, the proposed method (5) does not completely eliminate the

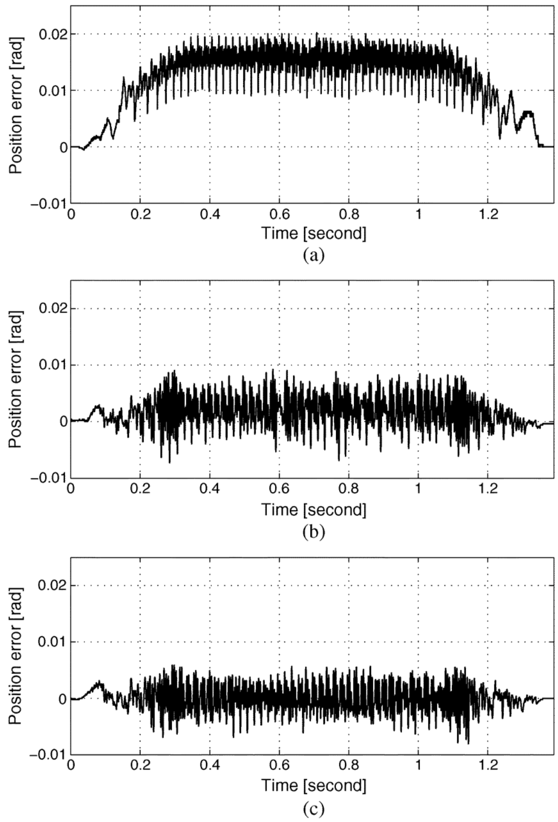

Fig. 5. Position-tracking error. (a) Case 1: Only microstepping (12) was used with current PI feedback. (b) Case 2: PID controller without the velocity feedforward (4) and the commutation scheme (11) were used with current PI feedback. (c) Case 3: PID controller with the velocity feedforward (5) and the commutation scheme (11) were used with current PI feedback.

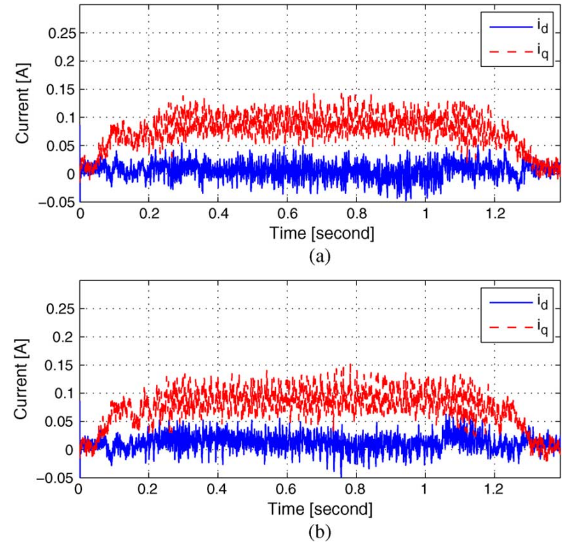

Fig. 6. i_q and i_q. (a) Case 2: PID controller without the velocity feedforward (4) and the commutation scheme (11) were used with current PI feedback. (b) Case 3: PID controller with the velocity feedforward (5) and the commutation scheme (11) were used with current PI feedback.

steady-state position-tracking error in Fig. 5(c). Fig. 6 shows the direct current i_d and the quadrature current i_q of cases 2 and 3. It was observed that zero direct currents of cases 2 and 3 were maintained although position ripples appeared. Therefore, the FOCs were achieved by the control methods that used the proposed commutation scheme (11) even though DQ transformation was not used.

IV. Conclusion

In this paper, a simple FOC without DQ transformation was proposed for the position tracking of PMSMs. The PID controller with velocity feedforward was developed for the torque modulation. The commutation scheme was proposed to generate the desired currents for the desired torque. The proposed desired currents are the same as a microstepping input which has a time-varying desired torque and the desired electrical position, which is always in advance of the present electrical position by $\pi/2$. The proposed method is in the form of an FOC without the use of DQ transformation. Experiments indicate that the proposed commutation scheme enables the PMSM to operate similar to a two-phase permanent magnet synchronous motor. And, the FOC was achieved by the proposed method even though DQ transformation was not used.

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