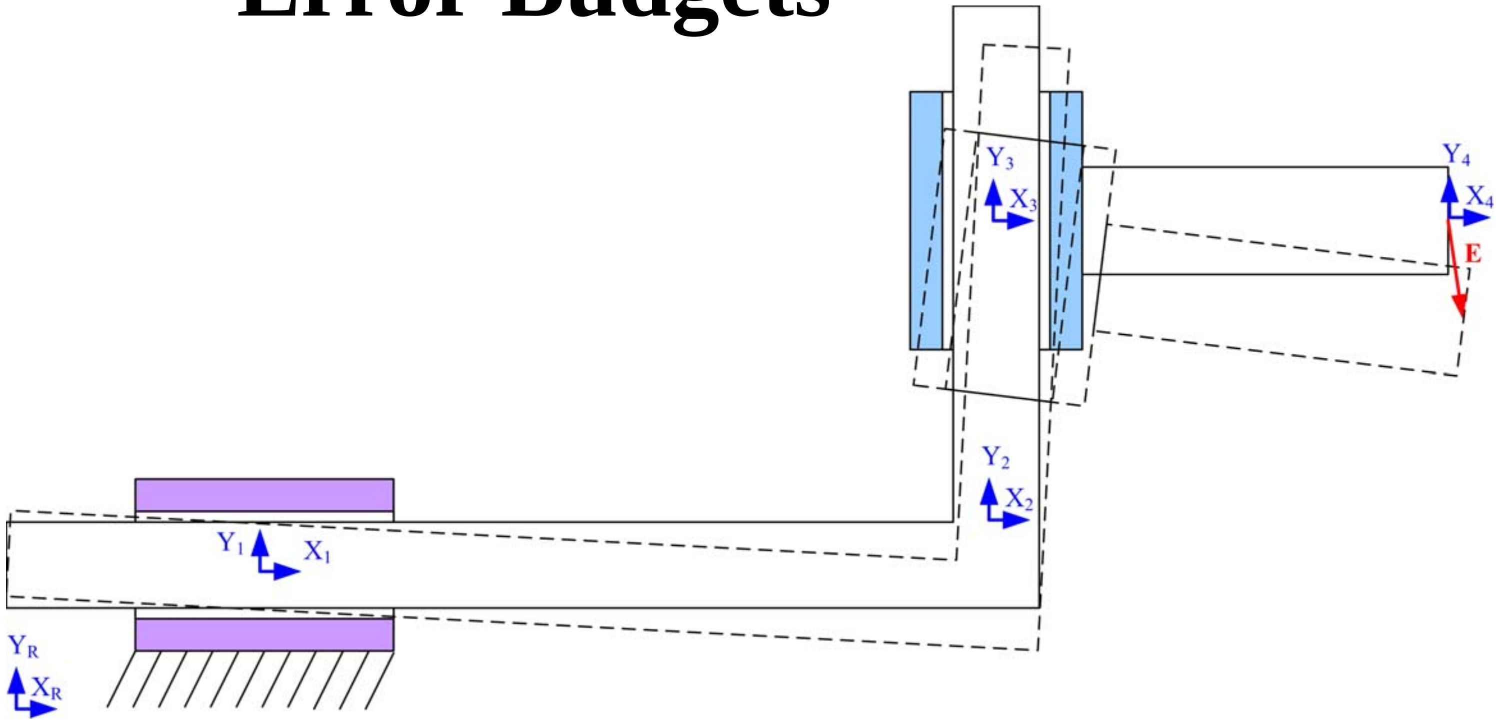
Topic 4
Error Budgets



Error Budgets

- A fast low cost tool to evaluate concepts before spending the time ($) to solid model and FEA (which will not catch geometric errors…), because:
  - Nothing is perfect
  - Need to predict accuracy and repeatability of machine
  - Need to better predict loads/life of bearings!
- Start with basics
  - Structural loop
  - Stick figures
  - Error Budget Spreadsheets
  - Homogeneous Transformation Matrices
  - Modeling Error Motions

Error Budgets: Design Strategy

- Four primary types of error include
  - Geometric
  - Load-induced
  - Thermal
  - Process
- For high precision machines, the magnitude of each will be about equal if there is a balanced allocation of resources
  - During the concept phase, develop the geometric-based error budget to be 4N better than required
    - Use simple Homogeneous Transformation Matrix-based spreadsheets
    - This lets you create the overall geometry (and spacing) of elements
  - Next, use solid models and FEA to ensure load-induced and thermal deflections are within limits
- Error budgets are not only useful for predicting the accuracy and repeatability of a machine, they are good for predicting misalignments loads on bearings

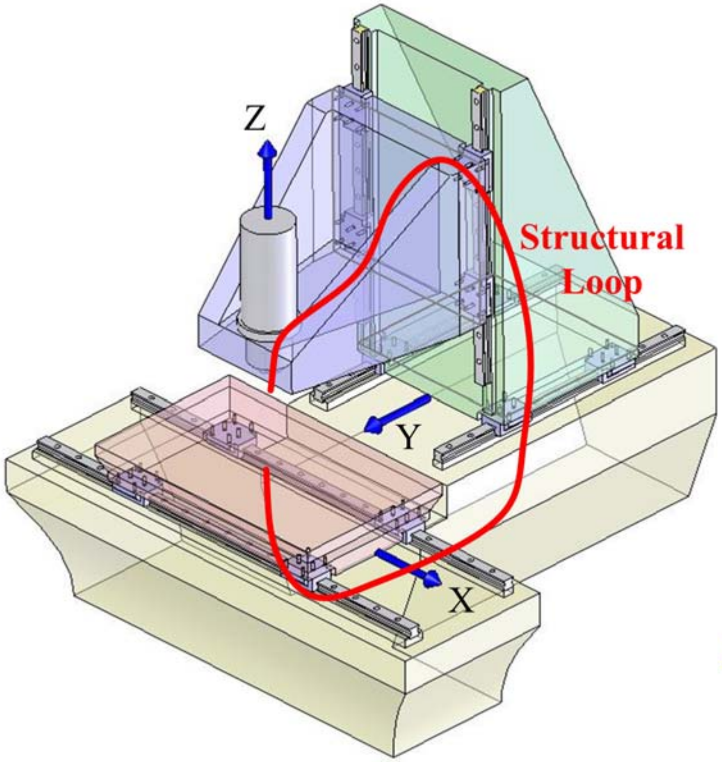
Error Budgets: Initial Allocations

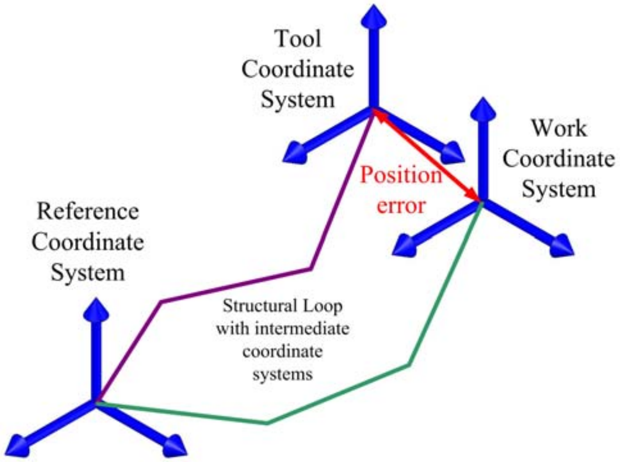
- The first issue with a precision machine is to understand the overall requirements:
  - Operating conditions
    - What are the dominant physics?
  - Accuracy, repeatability, resolution…
- Consideration of the dominant parameters enable 1st-order apportioning of error amongst axes & components
  - *This is a critical catalyst for creating viable strategies and concepts*
- This can be done for the RANDOM and Systematic cases!

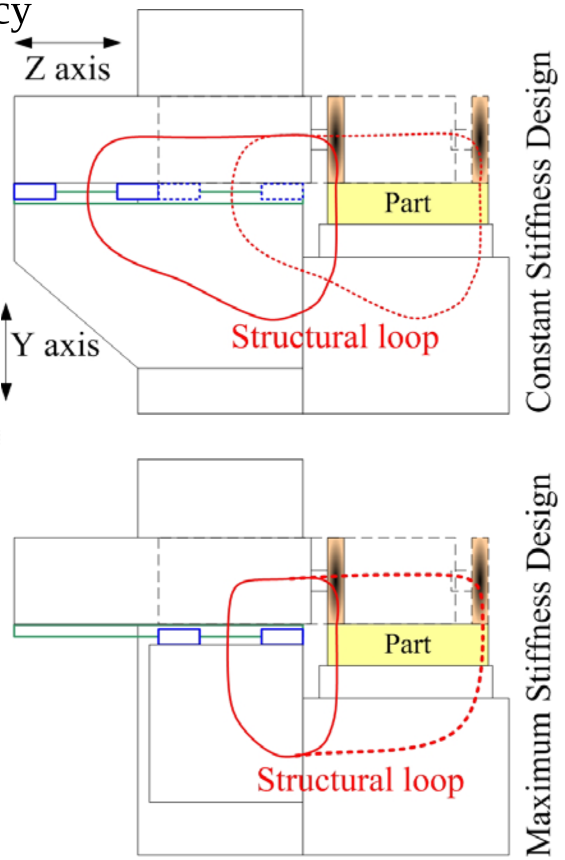
Axis_error_apportionment_estimator.xls								
To apportion errors between types and axes								
By Alex Slocum, last modified 10/12/06 by Alex Slocum								
Enter numbers in **BOLD,** Results in **RED**								
Number of axes	3	N						
Total allowable error (microns)	10	dtot						
				Apportion of error within each axis (amount allocated to X, Y, Z direction) TBD by sensitive directions				
				Bearings (fb)	Structure (fs)	Actuator (fa)	Sensor (fs)	Cables (fc)
Source of error	Factor (f)	Apportion of error (dtot/f)	Apportion of error per axis	1	1	1	1	0.2
Geometric (fg)	1	2.500	0.833	0.198	0.198	0.198	0.198	0.040
Thermal (ft)	1	2.500	0.833	0.198	0.198	0.198	0.198	0.040
Load-induced (deflection) (fl)	1	2.500	0.833	0.198	0.198	0.198	0.198	0.040
Process (fp)	1	2.500	0.833	0.198	0.198	0.198	0.198	0.040

Structural Loops (review)

- The *Structural Loop* is the path that a load takes from the tool to the work
  - It contains joints and structural elements that locate the tool with respect to the workpiece
  - It can be represented as a stick-figure to enable a design engineer to create a ***concept***
  - Subtle differences can have a HUGE effect on the performance of a machine
  - The *structural loop* gives an indication of machine stiffness and accuracy
    - *The product of the length of the structural loop and the characteristic manufacturing and component accuracy (e.g., parts per million) is indicative of machine accuracy (ppm)*
    - Long-open *structural loops* have less stiffness and less accuracy

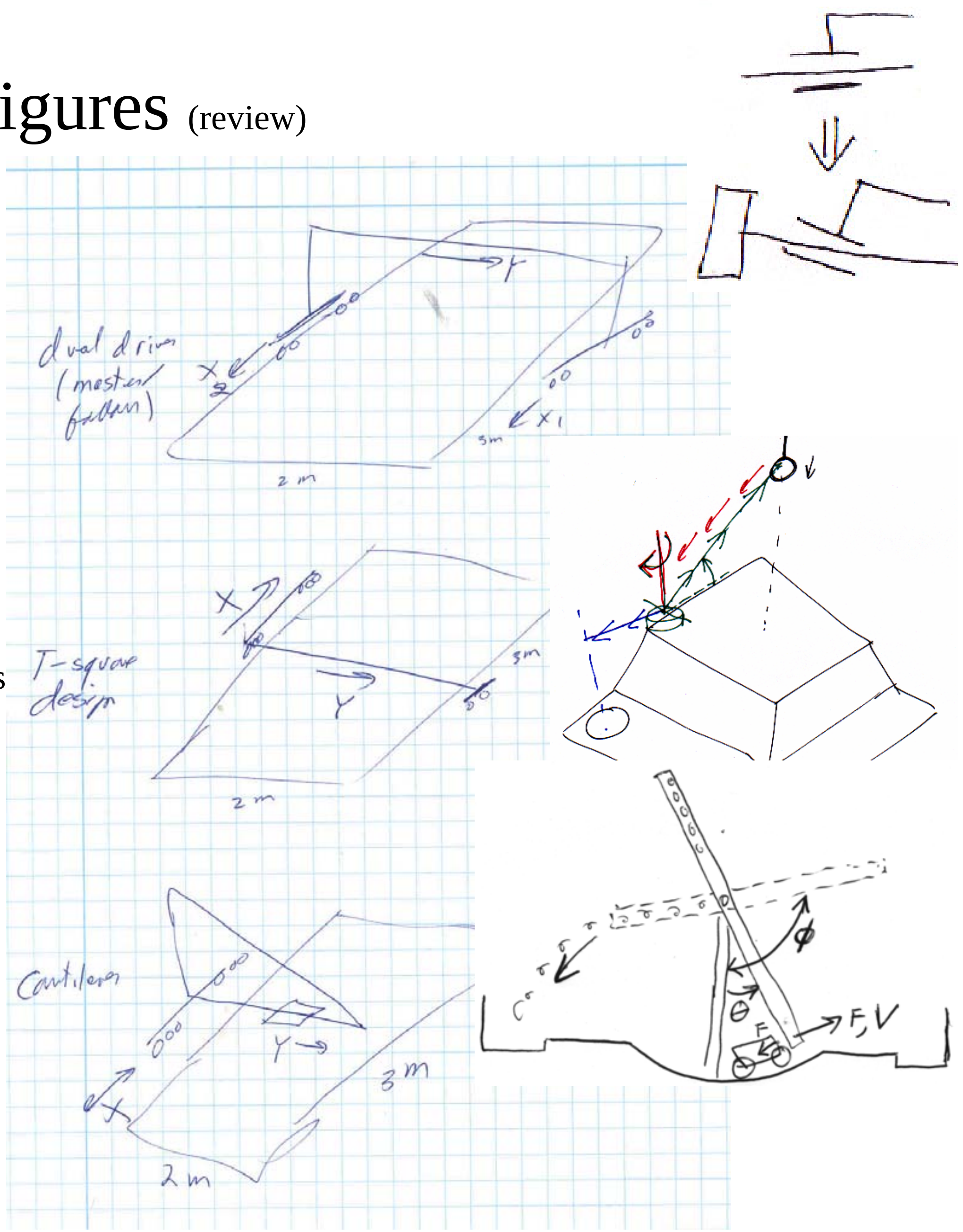






Stick Figures (review)

- Use of *fundamental principles* allows a designer to sketch a machine and error motions and coordinate systems just in terms of a *stick figure:*
  - The sticks join at centers of stiffness, mass, friction, and help to:
    - Define the sensitive directions in a machine
    - Locate coordinate systems
    - Set the stage for error budgeting
  - The designer is no longer encumbered by cross section size or bearing size
    - It helps to prevent the designer from locking in too early
- Error budget and preliminary load analysis can then indicate the required stiffness/load capacity required for each "stick" and "joint"
  - Appropriate cross sections and bearings can then be deterministically selected
- It is a "backwards tasking" solution method that is very very powerful!

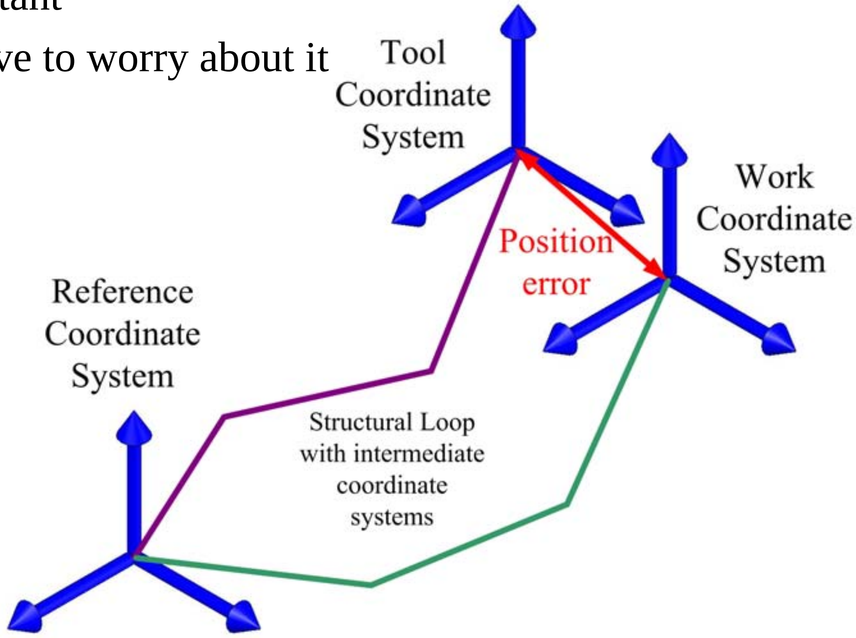


Error Budgets: *Homogeneous Transformation Matrices*

- HTMs help to model how motions in one link in a serial chain reflect through the chain

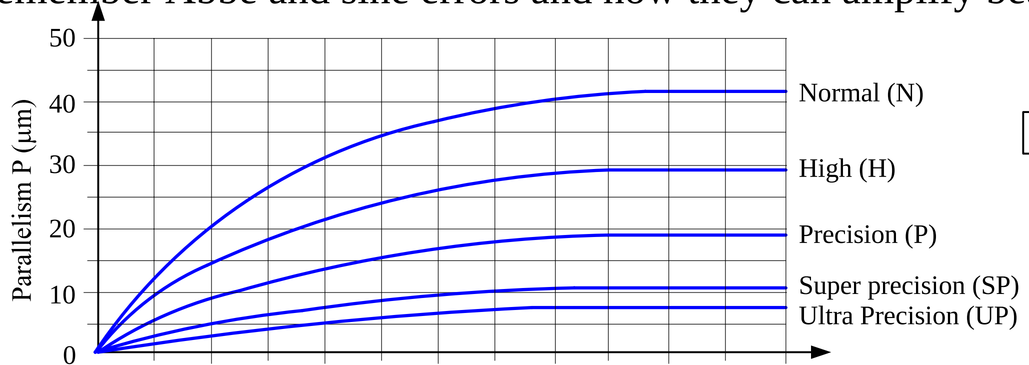
$$\begin{bmatrix} X_N \\ Y_N \\ Z_N \\ 1 \end{bmatrix} = {}^{N}T_{N+1} \begin{bmatrix} X_{N+1} \\ Y_{N+1} \\ Z_{N+1} \\ 1 \end{bmatrix} \qquad {}^{N}T_{N+1} = \begin{bmatrix} O_{ix} & O_{iy} & O_{iz} & P_x \\ O_{jx} & O_{jy} & O_{jz} & P_y \\ O_{kx} & O_{ky} & O_{kz} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

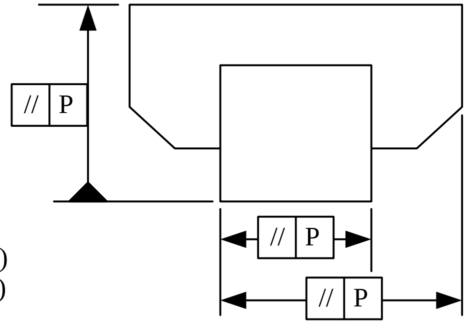
- Each column represents the direction cosines of the rotated axes
  - To avoid confusion about order of rotation, one HTM per axis of rotation
  - For small angular error motions, order not important
  - The spreadsheet does the math, so you do not have to worry about it
    - Just use good modeling technique

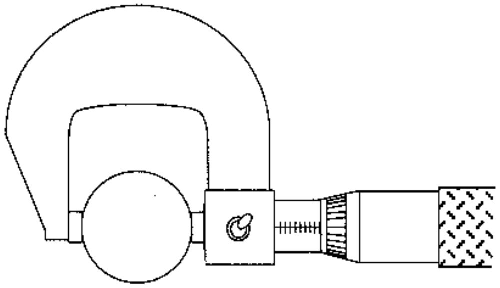


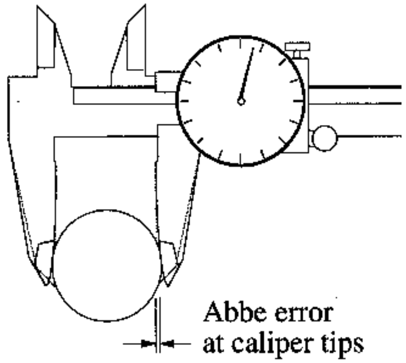
Error Motions

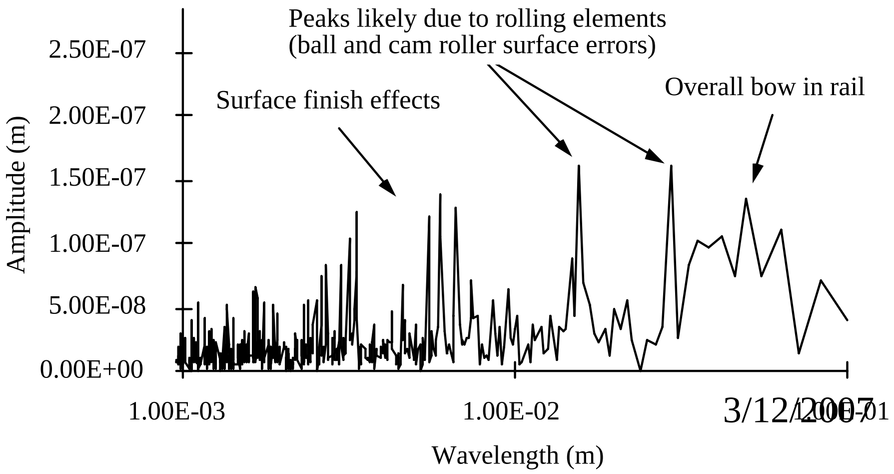
- Bearings are not perfect, and when they move, errors occur in their motion
  - Accuracy standards are known as *ABEC* (Annular Bearing Engineers Committee) or RBEC (Roller Bearing Engineers Committee) of the American Bearing Manufacturers Association (ABMA)
    - ABEC 3 & RBEC 3 rotary motion ball and roller bearings are common and low cost
    - ABEC 9 & RBEC 9 rotary motion ball and roller bearings are used in high precision machines
    - The International Standards organization (ISO) has a similar standard (ISO 492)
- An error budget is used to keep track of all the error motions in a machine
  - Remember Abbe and sine errors and how they can amplify bearing angular errors!





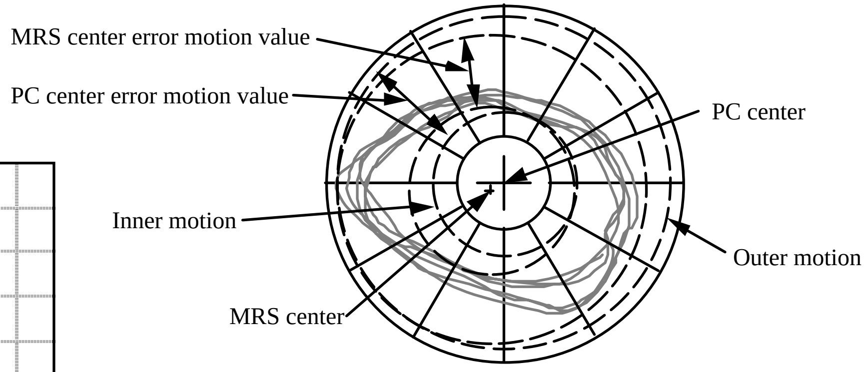




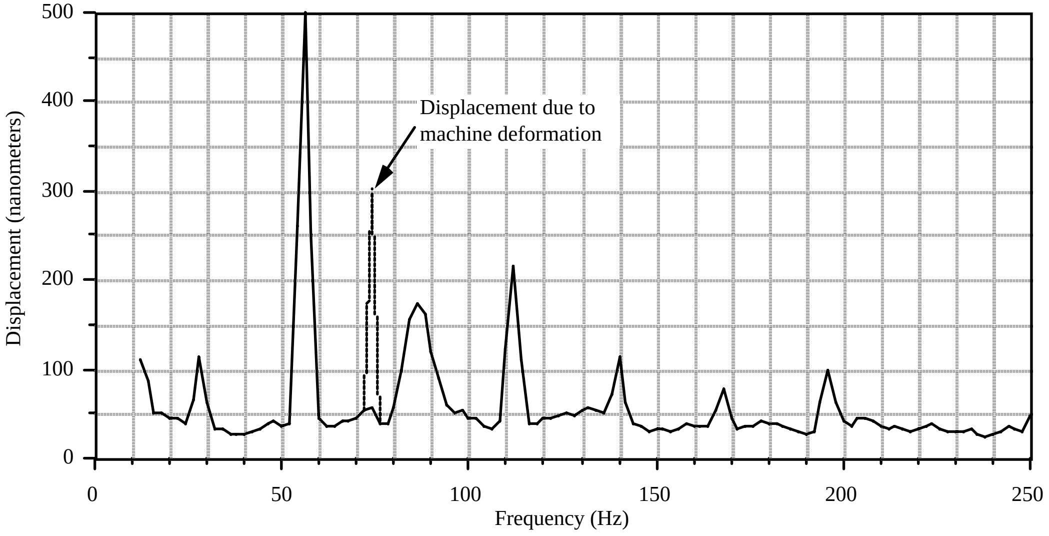


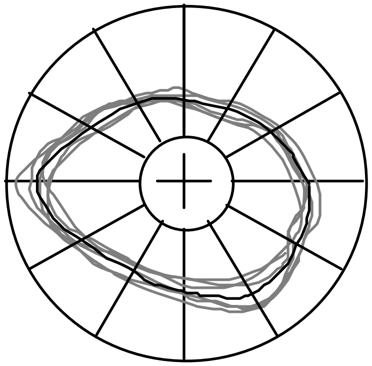
Error Motions: *Rotary Bearings*

- Standards exist for describing and measuring the errors of an *axis of rotation:*
  - Axis of Rotation: Methods for Specifying and Testing, ANSI Standard B89.3.4M-1985
- The digital age depends on hard disc drives which exist because of accurate repeatable rotary motion bearings
  - Radial, Axial, and Tilt error motions are of concern
- Precision Machine Designers measure error motions and use *Fourier transforms* to determine what is causing the errors…

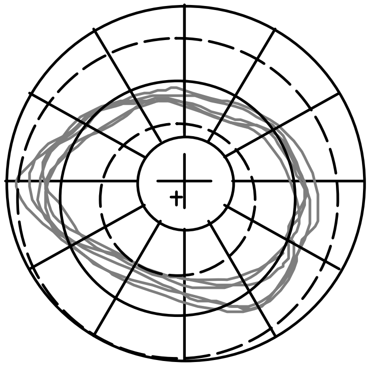


Total Error Motion





Average Error Motion



Fundamental Error Motion

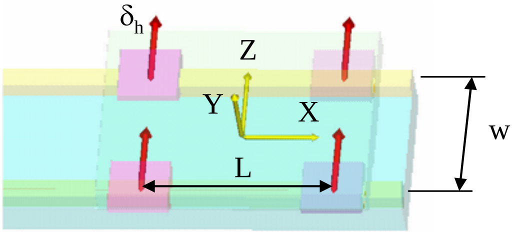
Error Motions: *Rotary Motion Estimates*

- Rotary bearings usually only come with an overall quality rating (e.g., ABEC 9, ISO 5)
  - The rating indicates ID and OD tolerance of the bearing
  - *The accuracy of the supported element (e.g., shaft) axis of rotation is usually dominated by the accuracy of the bore, shaft, alignment, and clamping method.*
    - Mel Liebers at Professional Instruments [MLiebers@airbearings.com] has tremendous insight on bearing measurement and mounting
      - As he points out, screw-actuated locknuts can also be used to preload a bearing and deform a shaft to correct for errors and thus achieve greater accuracy
        - » E.g. http://www.ame.com/
- As a first order estimate, assume the root square sum of the bore and shaft roundness are representative of the radial accuracy of the supported shaft.
  - Similar for axial accuracy
- Tilt accuracy can be estimated by radial accuracy divided by spacing between bearing sets
  - If just a single bearing set is used, tilt accuracy can be estimated by the flatness of the bearing mount (bore) divided by the bearing pitch diameter

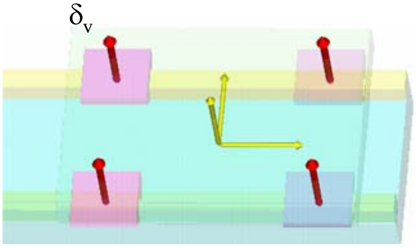
Error Motions: *Linear Bearings*

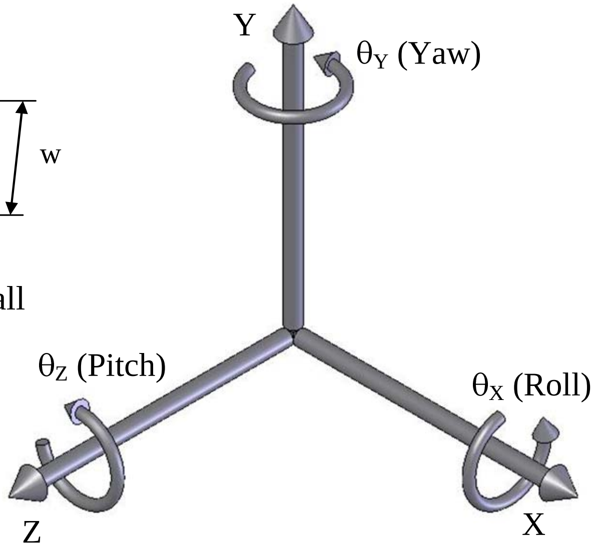
- The error motions of a carriage supported by a kinematic arrangement of bearings (exact constraint) can be determined "exactly"
- The error motions of a carriage supported by an elastically averaged set of bearings can be estimated by assuming the bearings act in pairs
- These calculations can be done using the “running parallelism” error information from the bearing supplier
  - The running parallelism number is usually a systematic (repeatable) error
  - The random error motion may typically be 10% of the running parallelism

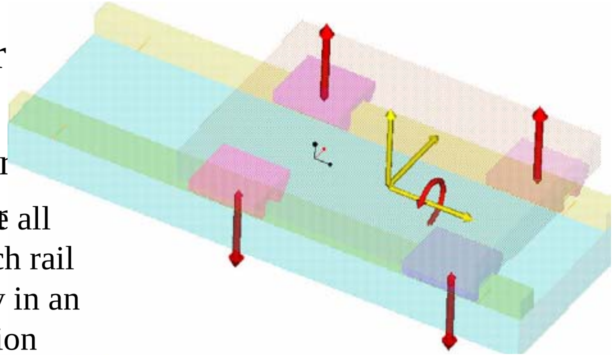
Horizontal Straightness: δ_y Assume all bearings move horizontally



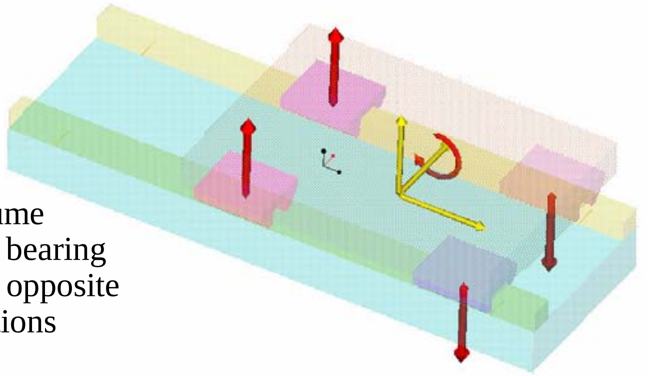
Vertical Straightness: δ_z Assume all bearings move vertically



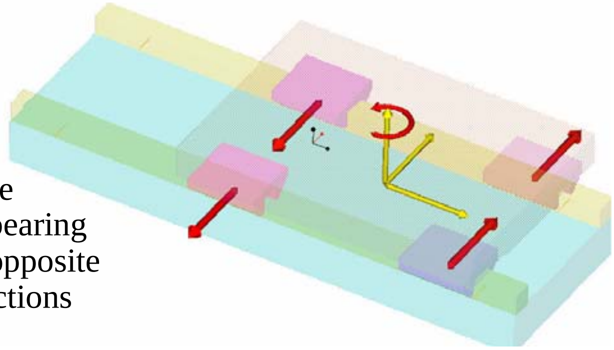




Roll: ε_x Assume all bearings on each rail move vertically in an opposite direction

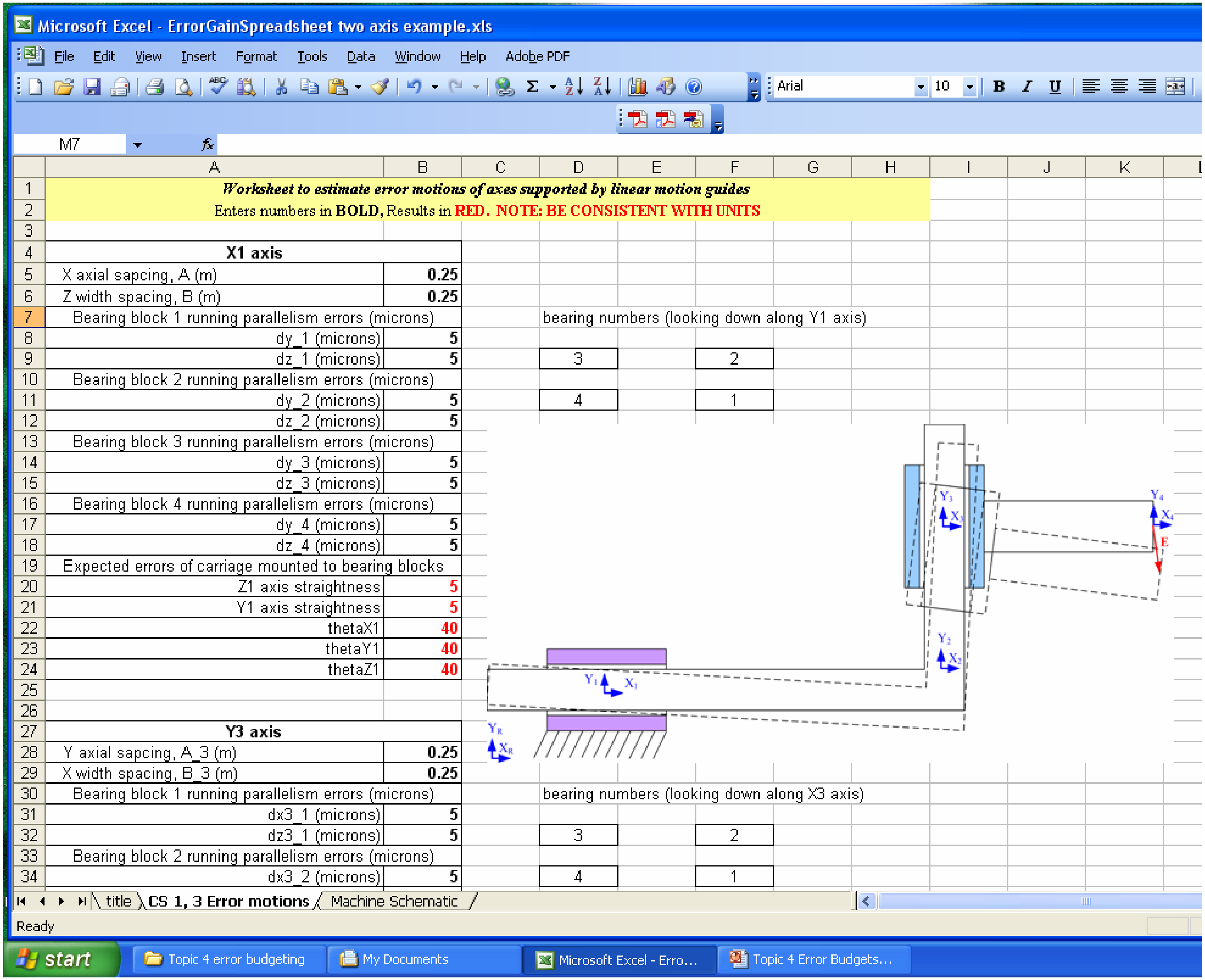


Pitch: ε_z Assume front and rear bearing pairs move in opposite vertical directions



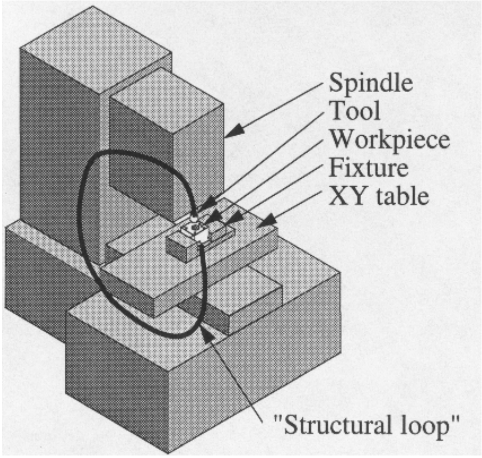
Yaw: ε_y Assume front and rear bearing pairs move in opposite horizontal directions

Error Motions: Linear Motion Estimates

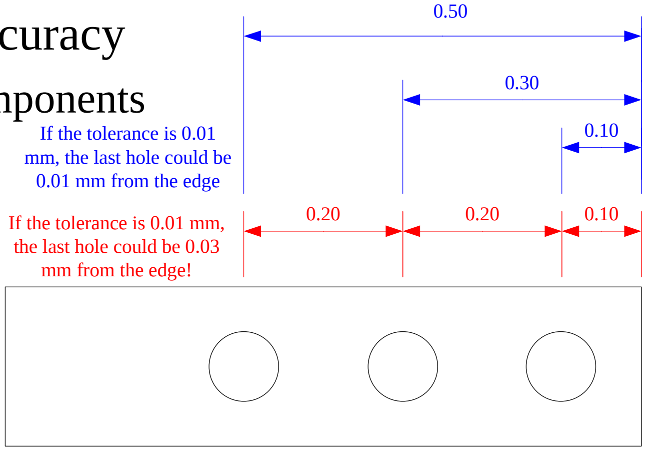


	A	B	C	D	E	F
1	***Worksheet to estimate error motions of axes supported by linear motion guides***					
2	Enters numbers in **BOLD,** Results in RED. NOTE: BE CONSISTENT WITH UNITS					
3						
4	**X1 axis**					
5	X axial sapcing, A (m)	**0.25**				
6	Z width spacing, B (m)	**0.25**				
7	Bearing block 1 running parallelism errors (microns)			bearing numbers (looking down along Y1 axis)		
8	dy_1 (microns)	**5**				
9	dz_1 (microns)	**5**		3		2
10	Bearing block 2 running parallelism errors (microns)					
11	dy_2 (microns)	**5**		4		1
12	dz_2 (microns)	**5**				
13	Bearing block 3 running parallelism errors (microns)					
14	dy_3 (microns)	**5**				
15	dz_3 (microns)	**5**				
16	Bearing block 4 running parallelism errors (microns)					
17	dy_4 (microns)	**5**				
18	dz_4 (microns)	**5**				
19	Expected errors of carriage mounted to bearing blocks					
20	Z1 axis straightness	**5**				
21	Y1 axis straightness	**5**				
22	thetaX1	**40**				
23	thetaY1	**40**				
24	thetaZ1	**40**				
25						
26						
27	**Y3 axis**					
28	Y axial sapcing, A_3 (m)	**0.25**				
29	X width spacing, B_3 (m)	**0.25**				
30	Bearing block 1 running parallelism errors (microns)			bearing numbers (looking down along X3 axis)		
31	dx3_1 (microns)	**5**				
32	dz3_1 (microns)	**5**		3		2
33	Bearing block 2 running parallelism errors (microns)					
34	dx3_2 (microns)	**5**		4		1

Errors: *Joints*

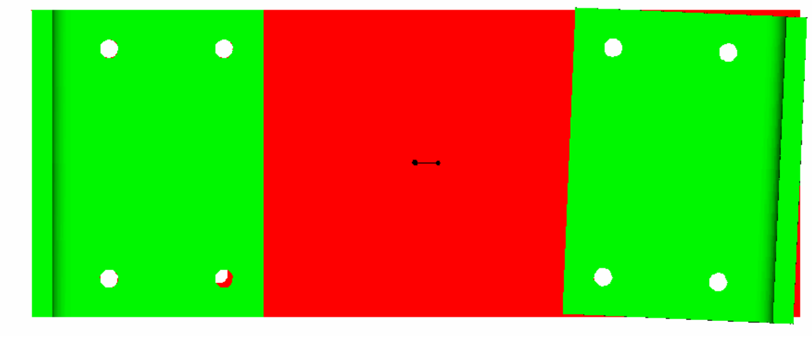


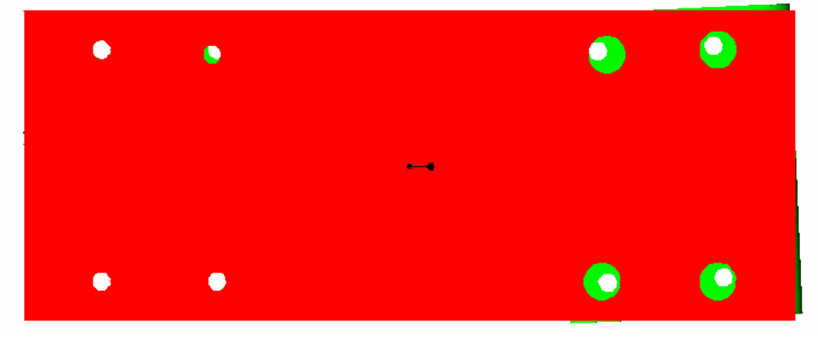
- Interfaces must enable parts to fit together with the desired accuracy
  - You cannot create two sets of exactly matching holes in two components
    - You can oversize the holes
    - The clearance between the bolts and holes means that the components do not have a unique assembly position
- "Error budgets" keep track of interferences & misalignments
  - These methods often assume "worst case tolerance"
  - For complex assemblies, advanced statistical methods are required
  - Deterministic designs are created using *financial*, *time*, and ***error*** budgets



If the tolerance is 0.01 mm, the last hole could be 0.01 mm from the edge

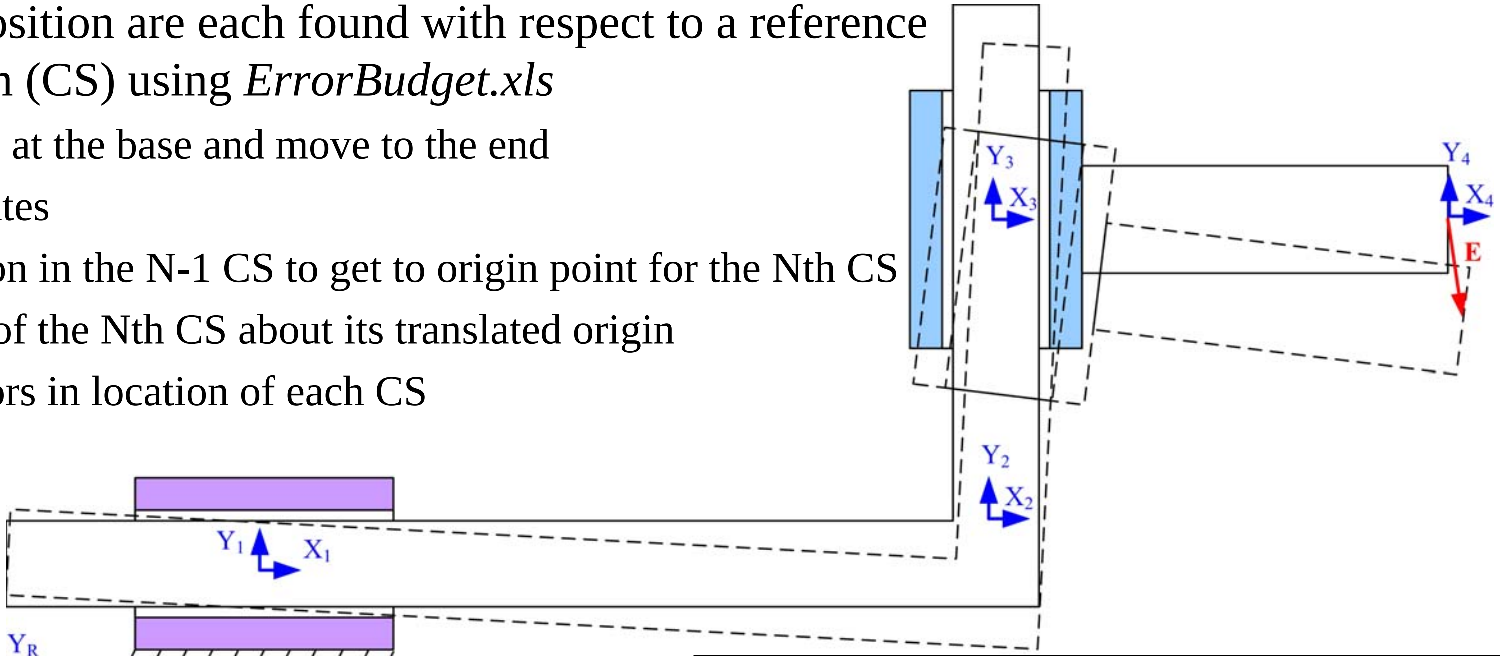
If the tolerance is 0.01 mm, the last hole could be 0.03 mm from the edge!



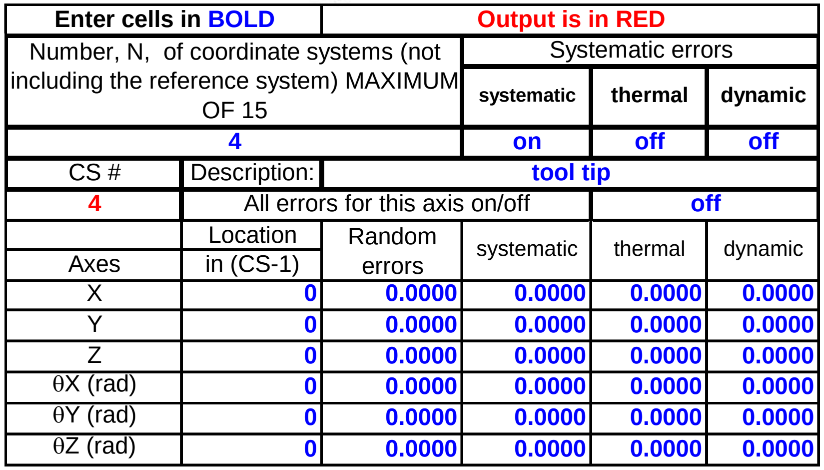


Error Budgets: *Spreadsheets*

- The accuracy and repeatability of a complex machine can be estimated using an error budget
  - An open chain (serial links) model with coordinate transformation matrices to model errors
    - Closed chain systems need to be approximated
- Tool and work position are each found with respect to a reference coordinate system (CS) using *ErrorBudget.xls*
  - Start with CS1 at the base and move to the end
  - Enter coordinates
    - Translation in the N-1 CS to get to origin point for the Nth CS
    - Rotation of the Nth CS about its translated origin
    - Enter errors in location of each CS



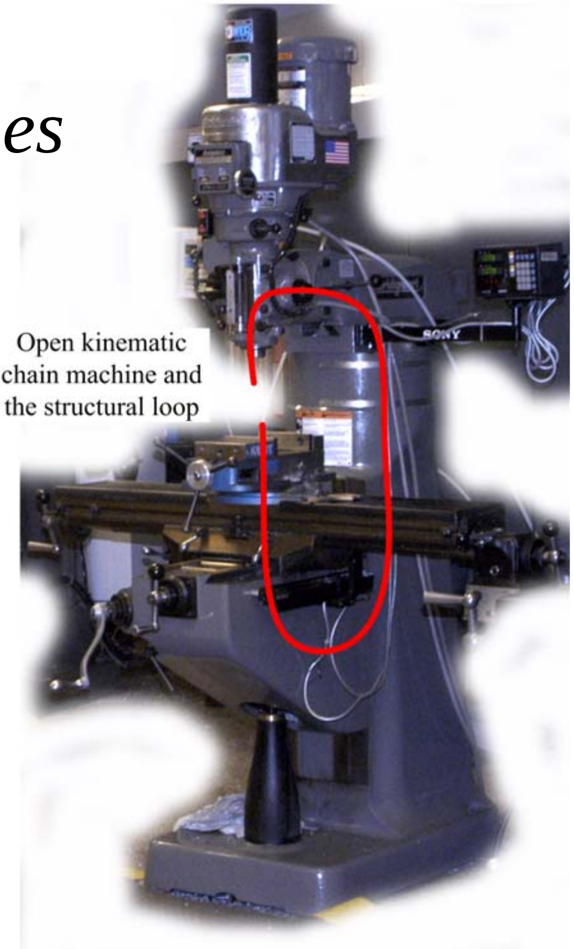
- Geometric (manufacturing) errors are easily modeled
- Deflections are more easily modeled using a solid model and finite element analysis
  - Deflections can then be added to geometric errors



Enter cells in **BOLD**		Output is in RED			
Number, N, of coordinate systems (not including the reference system) MAXIMUM OF 15			Systematic errors		
			systematic	**thermal**	**dynamic**
4			**on**	**off**	**off**
CS #	Description:	**tool tip**			
4	All errors for this axis on/off			**off**	
Axes	Location in (CS-1)	Random errors	systematic	thermal	dynamic
X	**0**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
Y	**0**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
Z	**0**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
θX (rad)	**0**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
θY (rad)	**0**	**0.0000**	**0.0000**	**0.0000**	**0.0000**
θZ (rad)	**0**	**0.0000**	**0.0000**	**0.0000**	**0.0000**

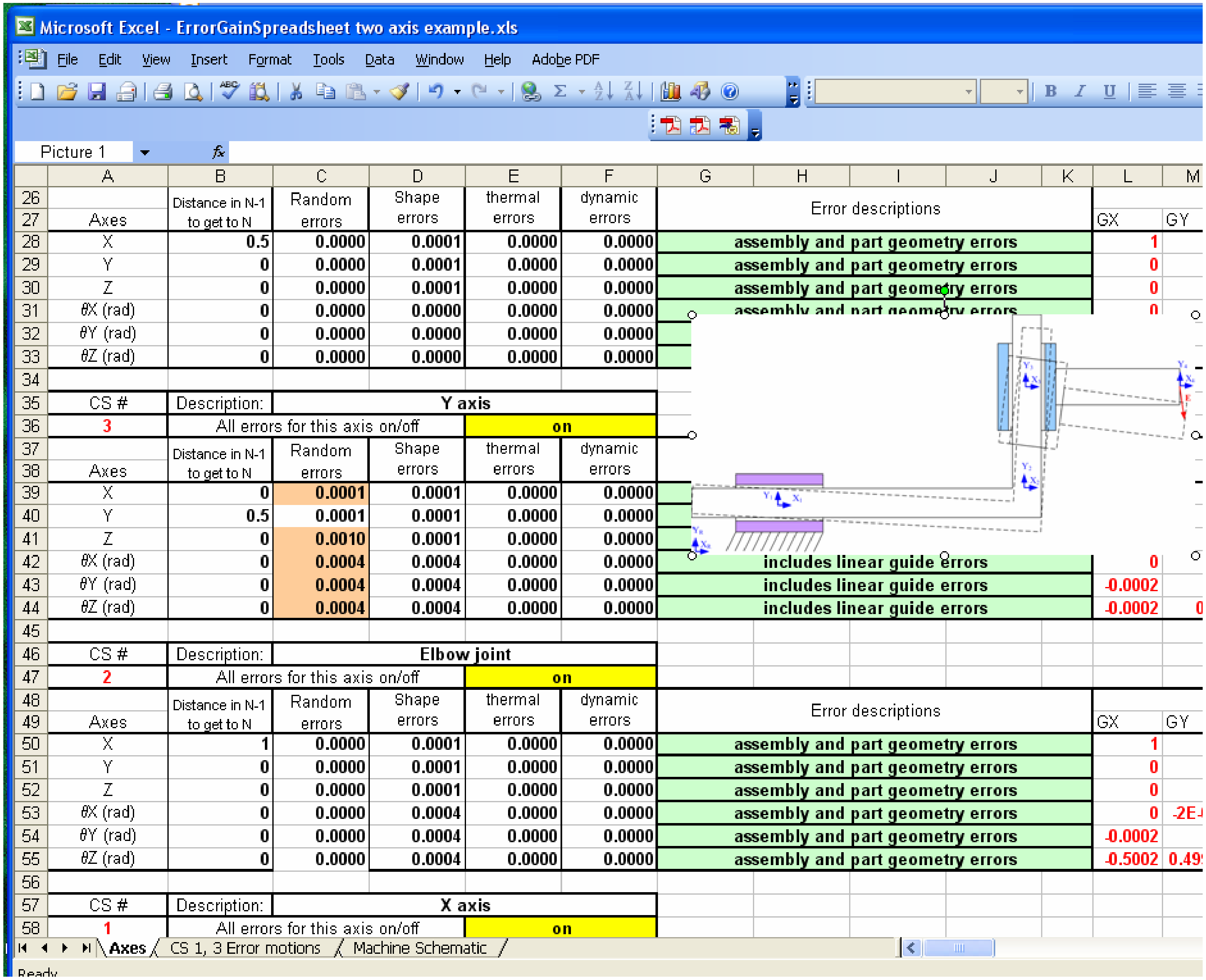
Error Budgets:
Spreadsheets & Modeling Techniques

- Open kinematic chain machines are straightforward to model
- Closed kinematic chains require local calculation of error motion or an equivalent open-chain model
  - E.g., kinematic coupling error motions
  - Bridge-type machines
    - Widely spaced bearings that simply support a bridge component are condensed to a single "very accurate" bearing that supports a cantilever
    - For example, the accuracy of this bearing is based on error motions of a carriage supported by bearings which are spaced the bridge-width apart


Open kinematic chain machine and the structural loop

ErrorGainSpreadsheet.xls
Written by Alex Slocum, John Moore, Whit Rappole October 4, 1999
Start with axis at the tool tip, and work back to the reference frame
Enter coordinates: displacements in the N-1 coordinate system to get to the origin of the current Nth coordinate system
Remember!!!!! The HTMs first translate, and THEN rotate. Rotational errors rotate the next (n+1) coordiante system !!!
The HTMs revert to identity matrices for coordinate systems beyond N, so do not need to delete entries where CS says "Not Used"
Enter numbers in BOLD, Results in **RED**

Error Budgets: *Spreadsheets & Modeling Techniques*



	A	B	C	D	E	F	G	H	I	J	K	L	M
26		Distance in N-1 to get to N	Random errors	Shape errors	thermal errors	dynamic errors	Error descriptions						
27	Axes											GX	GY
28	X	0.5	0.0000	0.0001	0.0000	0.0000	assembly and part geometry errors					1	
29	Y	0	0.0000	0.0001	0.0000	0.0000	assembly and part geometry errors					0	
30	Z	0	0.0000	0.0001	0.0000	0.0000	assembly and part geometry errors					0	
31	θX (rad)	0	0.0000	0.0000	0.0000	0.0000	assembly and part geometry errors					0	
32	θY (rad)	0	0.0000	0.0000	0.0000	0.0000							
33	θZ (rad)	0	0.0000	0.0000	0.0000	0.0000							
34													
35	CS #	Description:	Y axis										
36	3	All errors for this axis on/off			on								
37		Distance in N-1 to get to N	Random errors	Shape errors	thermal errors	dynamic errors							
38	Axes												
39	X	0	0.0001	0.0001	0.0000	0.0000							
40	Y	0.5	0.0001	0.0001	0.0000	0.0000							
41	Z	0	0.0010	0.0001	0.0000	0.0000							
42	θX (rad)	0	0.0004	0.0004	0.0000	0.0000	includes linear guide errors					0	
43	θY (rad)	0	0.0004	0.0004	0.0000	0.0000	includes linear guide errors					-0.0002	
44	θZ (rad)	0	0.0004	0.0004	0.0000	0.0000	includes linear guide errors					-0.0002	0
45													
46	CS #	Description:	Elbow joint										
47	2	All errors for this axis on/off			on								
48		Distance in N-1 to get to N	Random errors	Shape errors	thermal errors	dynamic errors	Error descriptions						
49	Axes											GX	GY
50	X	1	0.0000	0.0001	0.0000	0.0000	assembly and part geometry errors					1	
51	Y	0	0.0000	0.0001	0.0000	0.0000	assembly and part geometry errors					0	
52	Z	0	0.0000	0.0001	0.0000	0.0000	assembly and part geometry errors					0	
53	θX (rad)	0	0.0000	0.0004	0.0000	0.0000	assembly and part geometry errors					0	-2E
54	θY (rad)	0	0.0000	0.0004	0.0000	0.0000	assembly and part geometry errors					-0.0002	
55	θZ (rad)	0	0.0000	0.0004	0.0000	0.0000	assembly and part geometry errors					-0.5002	0.49
56													
57	CS #	Description:	X axis										
58	1	All errors for this axis on/off			on								

- Feel free to add worksheets to the workbook for computing errors in axes…
- Drag the sketch of the machine around for reference…
- Be VERY careful adding or deleting rows to the main spreadsheet!!!!!

Error Budgets: *Spreadsheets & Modeling Techniques*

- Remember the purpose of error budgeting is to enable you to try different things…

Two Axis Machine

Nominal coordinate totals		Sum Random Errors in the reference CS		RSS Random Errors in the reference CS		Average SUM & RSS random errors in the reference CS		Net Total Systematic Errors in the reference CS	
X=	1.6000	δX=	0.000426	δX=	0.000337	δX=	0.000382	δX=	-0.000001
Y=	0.6000	δY=	0.001085	δY=	0.000829	δY=	0.000957	δY=	0.001400
Z=	0.1000	δZ=	0.002210	δZ=	0.001572	δZ=	0.001891	δZ=	-0.000200
		εX (rad) =	0.000880	εX (rad) =	0.000622	εX (rad) =	0.000751	εX (rad) =	0.001200
		εY (rad) =	0.000880	εY (rad) =	0.000622	εY (rad) =	0.000751	εY (rad) =	0.001200
		εZ (rad) =	0.000880	εZ (rad) =	0.000622	εZ (rad) =	0.000751	εZ (rad) =	0.001200

Worksheet to estimate error motions of axes supported by linear motion guides
Enters numbers in **BOLD,** Results in **RED. NOTE: BE CONSISTENT WITH UNITS**

X1 axis	
X axial sapcing, A (m)	0.25
Z width spacing, B (m)	0.25
Bearing block 1 running parallelism errors (microns)	
dy_1 (microns)	5
dz_1 (microns)	5
Bearing block 2 running parallelism errors (microns)	
dy_2 (microns)	5
dz_2 (microns)	5
Bearing block 3 running parallelism errors (microns)	
dy_3 (microns)	5
dz_3 (microns)	5
Bearing block 4 running parallelism errors (microns)	
dy_4 (microns)	5
dz_4 (microns)	5
Expected errors of carriage mounted to bearing blocks	
Y1 axis straightness (m)	0.000005
Z1 axis straightness (m)	0.000005
thetaX1 (m)	0.00004
thetaY1 (m)	0.00004
thetaZ1 (m)	0.00004

bearing numbers (looking down along Y1 axis)

3	2
4	1