Optical Materials and Devices Problem Set

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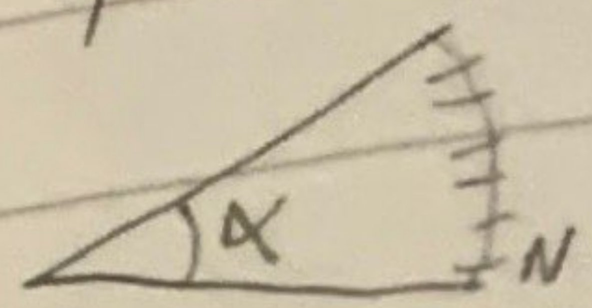
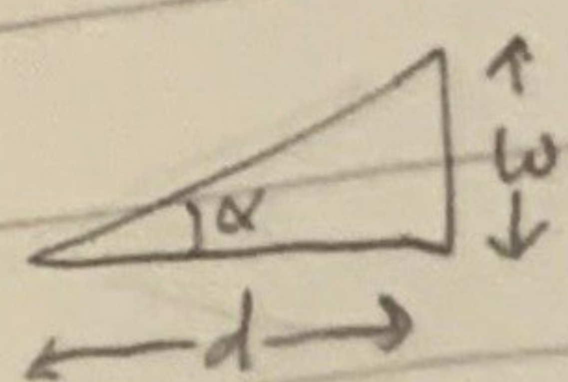
14.1a) $\frac{1000 \text{ lumens}}{200 \text{ lumens/W}} = 5W$ assuming an ideal source of white light

14.1b) spatial frequency: $\frac{N}{\alpha} = \frac{60 \text{ cycles}}{1^\circ} = 3438 \text{ cycles/rad}$

For $N=1$, $\alpha = 2.9 \cdot 10^{-4}$ rad





$\frac{w}{d} = \tan\alpha \approx \alpha \rightarrow w = \alpha d$

Plugging in: $w = (2.9 \cdot 10^{-4} \text{ rad})(0.5\text{m}) = 1.46 \cdot 10^{-4} \text{m}$

$\frac{1 \text{ dot}}{0.146\text{mm}} \cdot \frac{25.4\text{mm}}{1\text{in}} = 174 \text{ DPI}$

14.2a) $U = \frac{8\pi h \nu^3}{c^3 (e^{h\nu/kT} - 1)}$ from Equation 13.5

To find the max: $\frac{\partial U}{\partial \nu} = 0 = \frac{8\pi h}{c^3} \frac{3\nu^2 (e^{h\nu/kT} - 1) - \nu^3 \left(\frac{h}{kT} e^{h\nu/kT}\right)}{(e^{h\nu/kT} - 1)^2}$

$$0 = 3\nu^2 (e^{h\nu/kT} - 1) - \nu^3 \left(\frac{h}{kT} e^{\frac{h\nu}{kT}}\right)$$
$$0 = 3(e^{h\nu/kT} - 1) - \frac{h\nu}{kT} e^{h\nu/kT}$$
$$0 = 3(e^{h\nu/kT} - 1) - \frac{h\nu}{kT} e^{h\nu/kT}$$
$$3 = \left(\frac{h\nu}{kT} e^{h\nu/kT}\right) / \left(e^{h\nu/kT} - 1\right)$$
$$2.821 = \frac{h\nu}{kT} \rightarrow \nu = 2.821 \frac{kT}{h}$$
$$\lambda = \frac{c}{\nu} = \frac{hc}{2.821 kT}$$

$36^\circ C + 273.15 = 309.15K \rightarrow \lambda = \frac{(6.626 \cdot 10^{-34} \text{J/Hz})(3 \cdot 10^8 \text{m/s})}{2.821 \, (1.38 \cdot 10^{-23} \text{J/K})(309.15\text{K})}$

$\rightarrow \lambda = 16.5 \mu\text{m}$

At $2.74K \rightarrow \lambda = 1.86\text{mm}$

14.2b) red appears at $\lambda = 700nm$

$$T = \frac{ch}{2.821 k \lambda} \approx 7300K = 7026.85°C$$

14.2c) Stefan-Boltzman Law (Equation 13.7): $R = 5.67 \cdot 10^{-8} T^4 \frac{W}{m^2}$

Plug in $T = 309.15K$: $R = 5.67 \cdot 10^{-8} \cdot 309.15^4 = 517.9 \frac{W}{m^2}$

14.3a) $\begin{bmatrix} E_x' \\ E_y' \end{bmatrix} = e^{-i\sigma} \begin{bmatrix} e^{-i\delta} & 0 \\ 0 & e^{i\delta} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$ where $\sigma = (n_s + n_f) \frac{wd}{2c}$ &

$\delta = (n_s - n_f) \frac{wd}{2c}$

For 90° turn: $\left.\begin{aligned} E_x' &= -E_y = e^{-i\sigma} e^{-i\delta} E_x \\ E_y' &= E_x = e^{-i\sigma} e^{i\delta} E_y \end{aligned}\right\} \therefore |E_x| = |E_y|$ (must start at 45°)

For $|E_x| = |E_y| = A$: $\begin{aligned} -1 &= e^{-i\sigma} e^{-i\delta} \\ 1 &= e^{-i\sigma} e^{i\delta} \end{aligned} \Rightarrow -1 = e^{i2\delta}$

$2\delta = \pi(2a+1),\ a \in \mathbb{Z}$

From Quentin:

thinnest: $a = 0 \Rightarrow 2\delta = \pi$

$$\delta = \frac{\pi}{2} = (n_s - n_f) \frac{wd}{2c}$$

$$w = \frac{2\pi c}{\lambda} \Rightarrow \frac{\pi}{2} = (n_s - n_f) \frac{2\pi c d}{2c\lambda}$$

$$\Rightarrow \frac{1}{2} = (n_s - n_f) \frac{d}{\lambda}$$

$$d = \frac{\lambda}{2(n_s - n_f)}$$

For calcite: $n_s = 1.658$, $n_f = 1.486$

Plugging in: $d = \frac{600nm}{2(1.658 - 1.486)} = 1.744 \mu m$

↳ this ends up being a half wave plate (at higher multiples)

14.3b) Goal to design a quarter wave plate

14.3c) Now we want to rotate them.

I had trouble solving these properly.

14.4) $\varphi_x - \varphi_y = \pi = \frac{1}{c}\omega n_o^3 r_{63} V$ (Equation 13.30)

Plug in $\omega = \frac{2\pi c}{\lambda} \rightarrow \pi = \frac{2\pi c}{c\lambda} n_o^3 r_{63} V$

$$V = \frac{\lambda}{2 n_o^3 r_{63}} = \frac{700\,nm}{2(1.51)^3 (10.6 \cdot 10^{-3}\,\frac{nm}{V})}$$

$V = 9.59\,kV$ There are kW op-amps available now.

14.5a) Equation 13.34 (Bragg diffraction condition):

$$2\lambda_{phonon} \sin\theta = \frac{\lambda_{photon}}{n} \quad \text{where } \lambda_{phonon} = \frac{v_s}{V_s}$$

$$\theta = \arcsin\theta\left(\frac{\lambda_{photon} V_s}{2 v_s n}\right)$$

$$= \arcsin\left(\frac{700nm\,(100 \cdot 10^6\,Hz)}{2(7.4 \cdot 10^3\,m/s)(2.25)}\right)$$

$$= \arcsin(2.1 \cdot 10^{-3}) = 0.12°$$

14.5b) $\frac{I_{diff}}{I_{in}} = \sin^2\left(\frac{\pi \ell \sqrt{M I_a}}{\lambda\sqrt{2}}\right)$ (Equation 13.35)

$$I_a = \frac{1W}{(1mm)^2} = \frac{1MW}{m^2}$$

$$M = \frac{n^6 p^2}{\rho V_s^3} \xrightarrow{\text{Plug in:}} \frac{(2.25)^6 (0.15)^2}{(4700\frac{kg}{m^3})(7.4 \cdot 10^3\frac{m}{s})^3}$$

$$= 1.53 \cdot 10^{-15} \frac{m^2}{W}$$

$\ell = 1mm$

$$\Rightarrow \frac{I_{diff}}{I_{in}} = \sin^2\left(\frac{\pi (10^{-3}m)\sqrt{1.533 \cdot 10^{-15}(1 \cdot 10^6)}}{700 \cdot 10^{-9}\,m\sqrt{2}}\right)$$

$= 0.015$ (AO modulators are fast w/ no moving parts)

↳ faster than typical tiltable mirrors but not efficient at getting all the light through

14.5c) $N = \frac{\Delta V_s D}{v_s} \rightarrow \Delta V_s = \frac{N v_s}{D} = \frac{(1000)(7.4 \cdot 10^3\,\frac{m}{s})}{10^{-2}\,m}$

$= 740\,MHz$

acousto-optic modulators