Transducers Problem Set

05/01/2025

15.1) $E \approx 2E_F - 2E_c e^{-2/N_F V}$ (Equation 15.6)

Taylor Expansion: $E_T \doteq \sum_{k=0}^{\infty} E^k(0) \frac{V^k}{k!}$

$$E'(V) = -2E_c\left(\frac{-2}{N_F V^2}(-1)\right)e^{\frac{-2}{N_F V}} = -4\frac{E_c}{N_F V^2}e^{\frac{-2}{N_F V}}$$

$\lim_{V\to 0} E'(V) = 0$ $\therefore$ Taylor expansion fails around $V=0$.

15.2) $IV = mgv$ is measured by the Kibble balance

V: inverse AC Josephson effect (Eqn. 15.25): $V = n\frac{h}{2e}f$

I: quantum Hall effect (Eqn. 14.41): $R_H = \frac{1}{i}\frac{h}{e^2}$

Using Ohm's Law: $I = \frac{V}{R_H} = V \cdot \frac{ie^2}{h}$

Plug in V from above: $I = \left(\frac{nhf}{2e}\right)\frac{ie^2}{h} = \frac{nife}{2}$

Combine V & I in the Kibble balance equation:

$$IV = mgv \quad \& \quad IV = \frac{nife}{2}\left(\frac{nhf}{2e}\right) = \frac{n^2f^2ih}{4}$$

$$\therefore \quad mgv = \frac{n^2f^2ih}{4} \longrightarrow m = \frac{n^2f^2ih}{4gv}$$

(The basis for the 2019 redefinition of the kg)

★ Should be $n_V n_R i_R f_V f_R \frac{h}{4} = mgv$

15.3) Magnetic field from a wire given by Ampere's Law:

$$B(r) = \frac{\mu_0 I}{2\pi r} \xrightarrow[\text{is assumed to be constant}]{\text{through the SQUID's area}} \Phi = B(r)\cdot A = \frac{\mu_0 IA}{2\pi r}$$

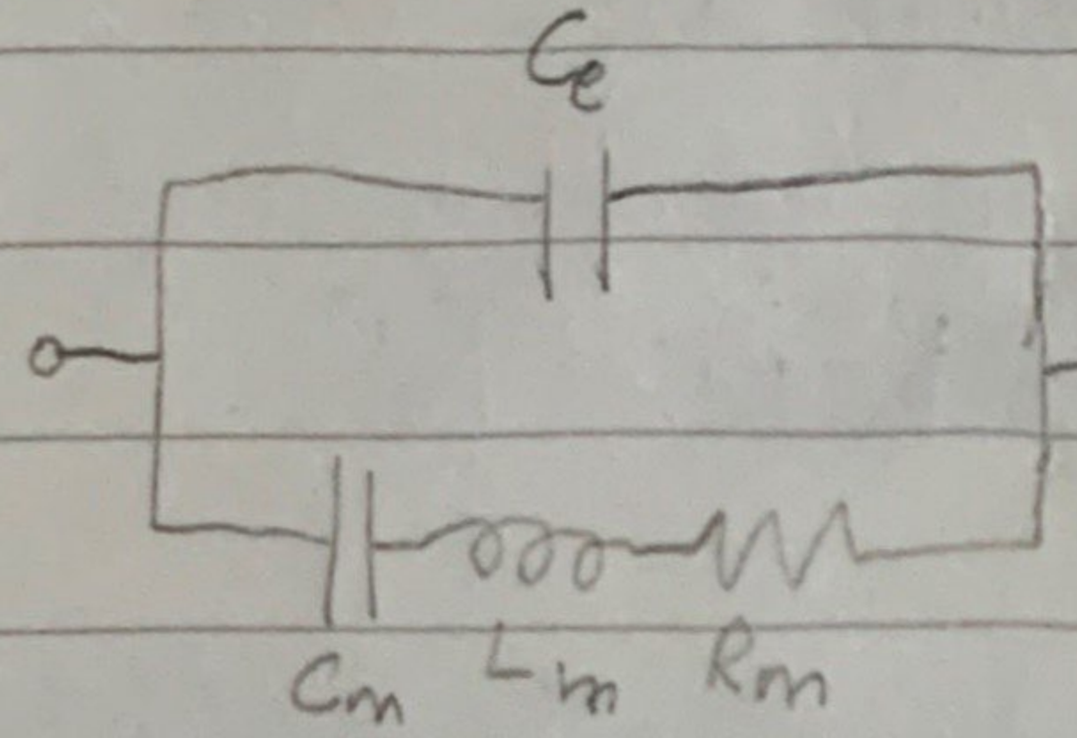
Plugging in $\Phi_0 = \frac{h}{2e} \approx 2.07\cdot 10^{-15}$ Wb since SQUID can detect one fluxon:

$$\Phi = \Phi_0 \Rightarrow \frac{\mu_0 IA}{2\pi r} = \Phi_0 \xrightarrow{\text{solve for } r} r = \frac{\mu_0 IA}{2\pi\Phi_0}$$

Plug in values: $r = \dfrac{(4\pi\cdot 10^{-7})(1)(1\cdot 10^{4})}{2\pi(2.07\cdot 10^{-15})} = 9660\,m = 9.66\,km$

$\therefore$ SQUID can detect the field of a 1A wire up to almost 10km away

X 15.4)



from Figure 15.4
(effective circuit for piezoelectric resonator)

impedance: $Z_m = Z_{C_m} + Z_{L_m} + Z_{R_m}$
$Z_m = \dfrac{1}{i\omega C_m} + i\omega L_m + R_m$ } from RLC circuit

$$Z = \frac{1}{\frac{1}{Z_e} + \frac{1}{Z_m}} = \frac{1}{i\omega C_e + \frac{1}{\frac{1}{i\omega C_m} + i\omega L_m + R_m}}$$

There is one resonant frequency in series and another in parallel.

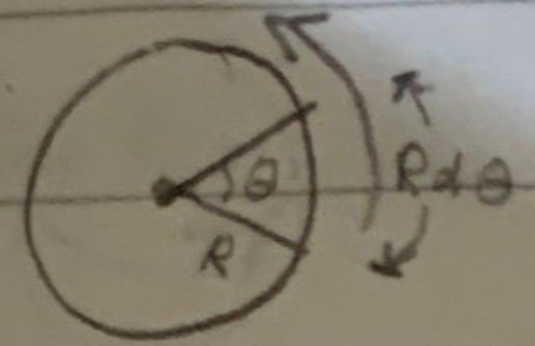
$\omega_{series} = \dfrac{1}{\sqrt{L_m C_m}} = 1.2910\cdot 10^{8}$

$\omega_{parallel} = \dfrac{1}{\sqrt{L_m \underbrace{\left(\frac{C_m C_e}{C_m + C_e}\right)}_{\text{should be } \frac{1}{\frac{1}{C_m}+\frac{1}{C_e}}}}} = 1.2935\cdot 10^{8}$

} Pluggin in values: $C_e = 5pF$, $C_m = 20fF$, $L_m = 3mH$, $R_m = 6\Omega$

I tried plotting in Mathematica but got stuck with it.

☆ Should review Quentin's solution which got it correct.

15.5 

Earth's radius

$$V = \frac{2\pi R}{1\,\text{day}} = \frac{2\pi(6.38\cdot10^{6}\,m)}{24\,hr\cdot\frac{3600\,s}{1\,hr}} = 464\,m/s$$

According to 15.3.3, Harrison's chronometer has a relative error of 10^{-5}.

$$(10^{-5}\,s)(30\,\text{days})\left(\frac{86400\,s}{\text{day}}\right)\left(\frac{462\,m}{s}\right) = 12\,km$$

According to the end of 15.3.1, Cs atomic clock has a relative error of 10^{-12}.

$$10^{-12}\,s\,(30\,\text{days})\left(\frac{86400\,s}{\text{day}}\right)\left(\frac{464\,m}{s}\right) = 1.2\,mm$$

15.6a) $V = \sqrt{\frac{GM}{r}}$ where $r = r_{earth} + r_{satellite}$

Plugging in: $V = \sqrt{\frac{6.67\cdot10^{-11}\,\frac{m^3}{kg\cdot s^2}\cdot5.97\cdot10^{24}\,kg}{20180\cdot10^{3}\,m + 6.37\cdot10^{6}\,m}} = 3873\,m/s$

how fast they travel

15.6b) $C = 2\pi r = 2\pi(20180\cdot10^{3}\,m + 6.37\cdot10^{6}\,m) = 1.2668\cdot10^{8}\,m$

orbital period

$$T = \frac{C}{V} = (1.2668\cdot10^{8}\,m)\left(\frac{1\,s}{3873\,m}\right) = 43072\,s \approx 12\,hrs$$

15.6c) From Eqn 15.55: $t_{earth} = \frac{t_{satellite}}{\sqrt{1-(\frac{v}{c})^2}} = \gamma\, t_{satellite}$

Solving for γ: $\gamma = \frac{1}{\sqrt{1-\frac{(3873)^2}{(3\cdot10^8)^2}}} = 1 + 8.34\cdot10^{-11}$

$\Delta t = t_{earth} - t_{satellite} = (\gamma - 1)\, t_{satellite}$

$= (8.34\cdot10^{-11})(43072\,s) = 3.6\,\mu s$

Since $t_{earth} > t_{satellite}$, the satellite's clock is slower

Eqn. 15.59:

15.6d) $t_{earth} = \dfrac{1-\frac{GM}{r_e c^2}}{1-\frac{GM}{r c^2}} t_{satellite} = \gamma_G T \Rightarrow \gamma_G = 1-5.276\cdot 10^{-10}$

$\Delta t = (\gamma_G - 1) t_{satellite} = -22.8 \mu s$

Since $t_{earth} < t_{satellite}$, the ground station's clock is slower

Since $|\gamma - 1| < |\gamma_G - 1|$, general relativity dominates.