PS1

2·1) a) yoctomole = 10^{-24} moles

6.022 140 76 ×10^23 atoms in 1 mole (avogadro's const.)

Ans. = (Avogadro) × 10^{-24}
= 0.6 atoms.

b) $10^{-9} \times (100) \times 365 \times 24 \times 3600$

GB, days/yr, hr, secs

= 3.1536 secs. similar to (A)

2·2) exabyte = 10^{18} bytes

Assuming 5 gb capacity and 1 mm thickness.

Stack height = $\frac{10^{18}}{10^{9}} \times 10^{-3}$ m

(10^9 → gb)

= 10^{6} m similar to 4×10^{5} m (to space)

2·3) 10^{80} atoms in the universe

largest number → $2^{10^{80}} - 1$

$2^{10} \approx 10^{3}$

2·4) $G = 6.673 \times 10^{-11}\ m^3\, kg^{-1}\, s^{-2}$

Earth Mass = 5.98×10^{24} kg

Radius = 6.378×10^{6} m

$\frac{GM}{R^2}$.

Obj. Mass = 1 kg

Obj. Radius = 1 m

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$$\text{Ratio} = 20 \log\left(\frac{\text{Earth } M / \text{Earth } R^2}{1/(1)^2}\right) = 20 \log\left(0.147 \times 10^{12}\right)$$

$$= \underline{223.34} \text{ dB}$$

2.5) a) 1 ton of TNT ⟶ 907.185 kg.

↳ Nitrogen

$N + N \longrightarrow N_2$ stable covalent bond releases energy

$\underline{6 \times 10^9 \text{ J}}$ in 1 ton

because of electrons moving to lower energy state.

b) 10,000 tons of TNT

↓

$6 \times 10^9 \times 10^4 \approx 6 \times 10^{13}$ $\underline{\text{Joules}}$.

1.602176462×10−19 J

$\times\, 235 \times$ MeV $\times \dfrac{6 \times 10^{23}}{235 \text{ g}}$ → 1 atom

$\Rightarrow 6 \times 10^{13}$ J.

$m \approx 620$ g

c) $E = mc^2$

$= 0.62 \times 9 \times 10^{16} \approx \underline{5 \times 10^{14} \text{ J}}$ Much larger.

2.6) a) $\lambda = h/p = h/mv$

weight ≈ 0.2 kg

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weight $\approx$ 0.2 kg

speed = 90-100 mph $\rightarrow$ 44 m/s.

$\lambda = 6.626 \times 10^{-34} / 8.8$ m. $\approx 10^{-34}$

Guesswork at this pt.

b) Mass of nitrogen =

speed $\Rightarrow$ Rms formula = $\frac{3RT}{M}$ = $3 \times \frac{8.314 \times \text{Room temp}}{\text{Molar mass}}$.

$\lambda = h/mv = \frac{h}{3 \times 8.314 \times 298} \times \frac{1 \text{ atom mass}}{\text{Molar mass}}$

$= \frac{h \times (\text{no. of atoms in 1 mole})}{7432.71}$ $= \frac{6.626 \times 10^{-34+23}}{7432}$

$= 0.89 \times 10^{-14}$ m

c) Typical distance btw molecules.

$PV = nRT$

$10^5 V = n(8.314)\,298$

$n = 1$ mole

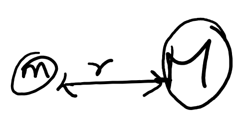
V_{mole} 0.02477 m^3

$V_{molecule} = 0.02477 / 6 \times 10^{23}$ $\Rightarrow$ $d = \sqrt[3]{V} \approx 3.3 \times 10^{-9}$ m

d) $T \rightarrow 298 \times 10^{-5}$

so that $\lambda \rightarrow$ similar order to d.

2.7) a) Potential is $-GMm/r$



$\frac{1}{2} m v^2 = \frac{GMm}{r}$

$\sqrt{2GM}$

Kinetic energy $\qquad v = \sqrt{\dfrac{2GM}{r}}$

b) $v = 3 \times 10^8$

$$9 \times 10^{16} = \frac{2GM}{r}$$

$$r = 2GM/c^2$$

c) $Mc^2 = hc/\lambda$

$$\lambda = h/cM$$

d) $\lambda = r \qquad \dfrac{h}{cM} = \dfrac{2GM}{c^2}$

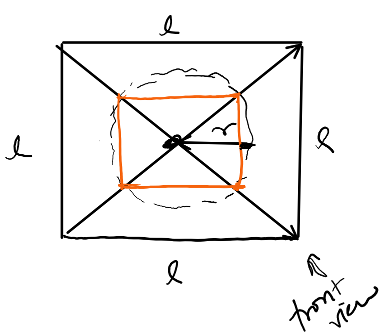
$$M = \sqrt{\frac{h}{1}\frac{c}{2G}}$$

e) f) g) $E = Mc^2$

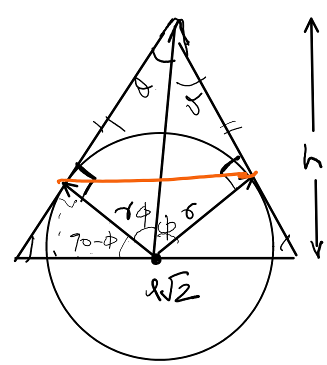
$$E = h\nu = \frac{h}{T}$$

2.8) Assuming Tangential to edges other than sq.

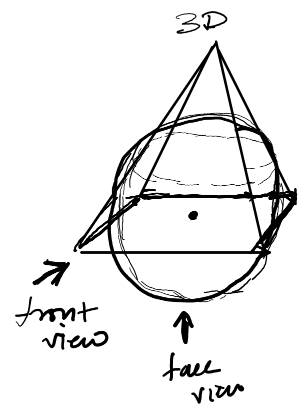
Top View:



Front View:



3D



$\sin\theta = r/h$

$\cos\phi = r/h \longrightarrow \quad \sin^2\phi + \cos^2\phi = 1$

$$\frac{r^2}{h^2} + \frac{2r^2}{l^2} = 1 \rightarrow r^2(l^2 + 2h^2) = l^2h^2$$

$\cos(90-\phi) = 2r/l\sqrt{2}$

$\sin\phi = \nearrow$

$$r = \frac{lh}{\sqrt{l^2+2h^2}} \rightarrow ①$$

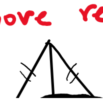
from ① $h^2(l^2 - 2r^2) = r^2l^2$

$$h = \frac{rl}{\sqrt{l^2-2r^2}}$$

h & l can be exclusive of each other

face view

remove red paths for vol.



$$h = \frac{rl}{\sqrt{l^2-2r^2}}$$

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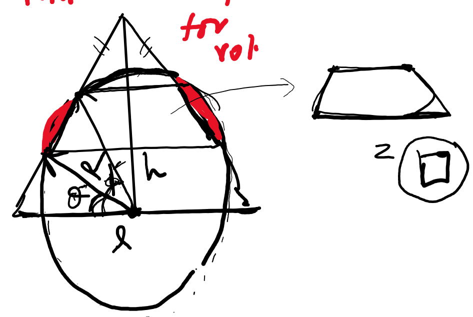
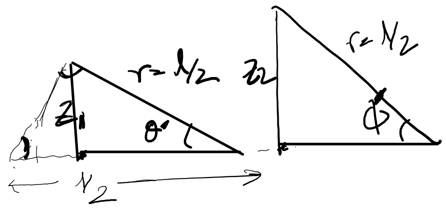


$$2\,\square$$

Volume of semisphere $= \int_0^R \pi(R^2 - z^2)\,dz$

$$= \frac{2\pi R^2}{3}$$



if we calculate $z_1, z_2 \rightarrow$ volume of common space

$$\int_0^{z_1} \pi(R^2 - z^2)\,dz + \int_{z_2}^{R} \pi(R^2 - z^2)\,dz + \int_{z_1}^{z_2} \text{area of square}\cdot dz\,.$$

$$\square$$

$$\pi\left(R^2 z - \frac{z^3}{3}\right)\Big|_0^{z_1} + \pi\left(R^2 z - \frac{z^3}{3}\right)\Big|_{z_2}^{R} + \square$$

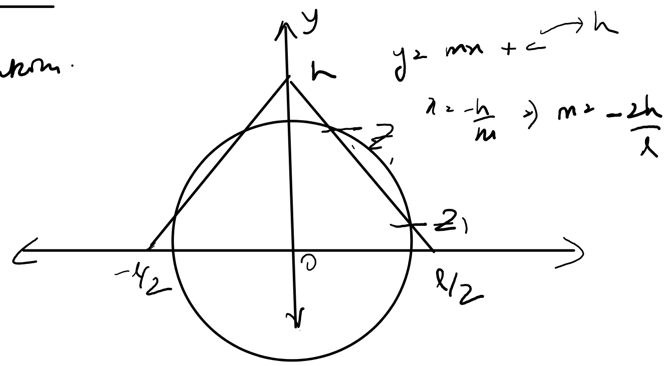
$$\pi R^2 z_1 - \frac{z_1^3}{3} + \frac{2\pi R^3}{3} - \pi R^2 z_2 + \frac{z_2^3}{3} + \square$$

finding z_1, z_2 using cartesian equation.

$x^2 + y^2 = r^2$ – circle

$y = -\frac{2h}{l}x + h$ – line

$x = -\frac{y-h}{2h} \times l$



$$(y-h)^2 \frac{l^2}{4h^2} + y^2 = \frac{l^2h^2}{l^2+4h^2} \rightarrow (l^2y^2 + h^2l^2 - 2hyl^2 + 4h^2y^2)(l^2+4h^2) = 4l^2h^4$$

$$y^2(l^4 + 4h^2l^2 + 4h^2l^2 + 16h^4) + y(-2hl^4 - 8h^3l^2) + h^2l^4 \cancel{+4h^4l^2} = \cancel{4l^2h^4}$$

solutions for y are z_1, z_2 $\Rightarrow$ $$\frac{(2hl^4 + 8h^3l^2) \pm \sqrt{(2hl^4+8h^3l^2)^2 - 4h^2l^4(l^4 + 8h^2l^2 + 16h^4)}}{2(l^4 + 8h^2l^2 + 16h^4)}$$

Plugin z_1, z_2 values into eq -

formula for $\square \longrightarrow$ $$\int^{z_2} (R^2 - z^2) \times 4\,dz = 4R^2(z_2 - z_1) - 4\left(\frac{z_2^3}{3} - \frac{z_1^3}{3}\right)$$

formula for $\boxed{\square} \longrightarrow \int_{z_1}^{z_2} (R^2 - z^2) \times 4 \, dz = 4R^2(z_2 - z_1) - 4\left(\frac{z_2^3}{3} - \frac{z_1^3}{3}\right)$

final answer seems to be a very long calculation

→ The correct problem (a subset of above problem).

$r = \frac{lh}{\sqrt{l^2+2h^2}}$ R is also $= l/2$

$\Rightarrow \frac{l}{2} = \frac{lh}{\sqrt{l^2+2h^2}}$

$l^2 + 2h^2 = 4h^2$

$l = \sqrt{2}h$

$h = l/\sqrt{2}$

Now $z_1 = 0$

$z_2 = \left(\frac{2hl^4 + 8h^3l^2}{l^4 + 8l^2h^2 + 16h^4}\right) \cancel{\times 2}$ $\stackrel{l = \sqrt{2}h}{=} \frac{2h(4h^4) + 8h^3(2h)}{4h^4 + 8h^2(2h) + 16h^4}$

$\frac{l^2}{2} - \frac{l^2}{6}$

$\frac{4l^2}{12}$

$\frac{2l}{\sqrt{3}}$

Now using z_1 & z_2 in the big volume calculation formula with integrals

$z_2 = \frac{24h}{28} = \frac{6}{7}h$

$\frac{2\pi R^3}{3} - \pi R^2 z_2 + \frac{z_2^3}{3} + 4R^2 z_2 - 4z_2^3$

$\frac{2\pi h^3}{3\sqrt{2}} - \pi h^3 \times \frac{3}{7} + \frac{12h^3}{7} - \frac{11}{3} \times \left(\frac{6}{7}\right)^3 \times h^3$

Simplify for volume