13.1) a) DIAMAGNETIC SUSCEPTIBILITY

eq. 13.15: $x_M = -\mu_0 \frac{q^2 Z r^2}{4 m_e V}$

$V = \frac{4}{3}\pi r^3$ r = Bohr radius $= 5.29 \cdot 10^{-11}$ m

where:

μ_0 = permeability freespace

q = charge of an electron

Z = electron scale factor ?

r = Bohr radius

M_e = mass of an electron

V = atom volume

$$x_m = -1.26e^{-6}\left[\frac{(1.6e{-19})^2 \cdot 1 \cdot (5.29e{-11})^2}{4 \cdot 9.1e{-31} \cdot (5.29e^{-11})^3}\right] \approx -10^{-5}$$

13.1) b)

eq. 12.7: $F = -V \mu_0 x_m H \frac{dH}{dz}$

integrate over z

where:

V = volume

μ_0 = ...

x_m = diamag susceptibility

H = mag field required

$\frac{dH}{dz}$ = gradient of field over volume

$$F = -V \mu_0 x_m \frac{H^2}{z}$$

$$H = \left(\frac{-Fz}{V \mu_0 x_m}\right)^{1/2}$$

$$H = \left(\frac{-0.1\,kg \cdot 9.8\,m/s^2 \cdot 0.1\,m}{10^{-4}\,m^3 \cdot 1.26e{-6} \cdot -10^{-5}}\right)^{1/2} = 3 \cdot 10^6\ A/m$$

$$H = \frac{B}{\mu_0} \therefore B = 3.7\ T$$ (larger than pm field)

13.2)

m for an electron is:

Bohr Magneton $m = \mu_B = \frac{e\hbar}{2m_e}$ which in SI units is $9.27e^{-24}$

solving for $\vec{B}$

$$B = \frac{\mu_0}{4\pi}\left(\frac{3\hat{x}(\vec{m}\cdot\hat{x}) - \vec{m}}{|x|^3}\right)$$

where $\hat{x}$ is the unit vector

$$= \frac{1.26e^{-6}}{4\pi}\left(\frac{3(9.27e{-24}) - 9.27e{-24}}{r^3}\right)$$

this would get huge at 1 angstrom

$$= 3e{-7}\left(\frac{18.54e{-24}}{r^3}\right) = \frac{5.4\,e^{-30}}{r^3}$$

definitely wrong

for electrostatic:

$$E_e = qE = q\frac{q}{4\pi\epsilon_0 r} = \frac{(1.6e{-19})^2}{4\pi\cdot 8.85e^{-12}\cdot r}$$

$$= \frac{2.302e{-28}}{r}$$

for 1 Angstrom $r = 10e{-10}$

$2.302e{-19}$

13.3) a) Electromagnetic energy density is

$$U = \frac{1}{2}\left(\cancel{\vec{E}\cdot\vec{D}} + \vec{B}\cdot\vec{H}\right) = \frac{1}{2} B \cdot H$$

no electrostatic contribution

$$U = \frac{1}{2\mu}\int B^2\, dV$$

μ in a ferromagnetic material is much greater than in air, so U is minimized when μ is maximized. The force is a function of this minimization.

13.3) b) For a permanent magnet

$$U = \frac{1}{2}\int \vec{B}\cdot\vec{H}\, d\vec{V} \qquad \text{where } H = \frac{1}{\mu_0}\vec{B} - \vec{M}$$

so $$U = \left(\frac{1}{2}\int \frac{1}{\mu_0}\vec{B}^2 - \vec{M}\cdot\vec{B}\right) d\vec{V}$$

The dot product $\vec{M}\cdot\vec{B}$ will be maximized if the two vectors are parallel, resulting in lower energy. (Flux from one with mag. vector of the other)

13.4) saturation for Fe @ 0K

from 13.21: $M = \frac{m}{V} = \mu_B \Delta n = \mu_B^2 B n(E_F)$

induced moment / volume (molar mass) — spin magnetic moment — fermi energy

$$\frac{6.02e23 \text{ atoms}}{55.85 \text{ g}} \cdot \frac{7.86 \text{ g}}{1 \text{ cm}^3} \cdot \frac{100^3 \text{ cm}}{1 \text{ m}} \cdot \frac{9.28e{-24} \text{ J/T}}{1 \text{ electron}}$$

$= 7.8e5$ Amps/meter ← this is an M

$M = \frac{1}{\mu} B \qquad \mu_{iron} = 6.3e{-3}$ H/m

$B = 7.8e5 \frac{A}{m} \cdot 6.3e{-3} \frac{H}{m}$

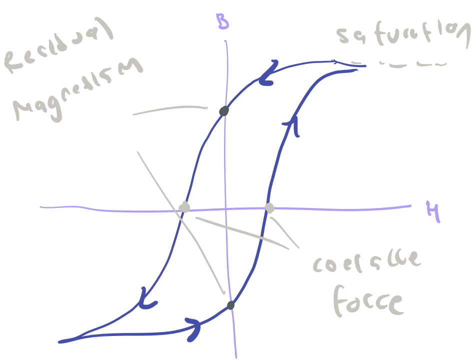
$= 49.14e2$ HA/m² ≈ 5000 T

(definitely wrong)

for this problem we are assuming saturation occurs when the spin of every electron's spin is aligned. Fe has 2 free valence electrons

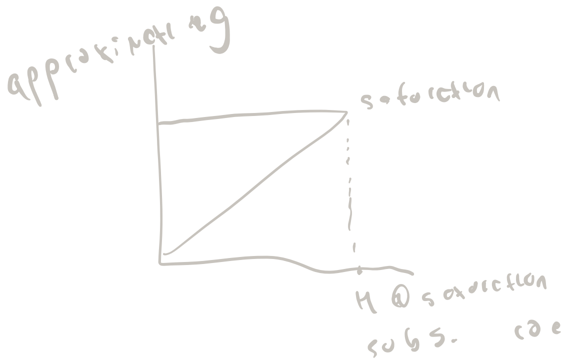
so

13.5) a)



b) find area of curve above:

approximating as:



saturation of iron

$\approx 2T$

coercivity of iron given $= 4 \cdot 10^3$ A/m

$$\text{energy/cycle} = \left(\frac{1}{2}\ 2T \cdot 4e3\ A/m\right) \cdot 2$$

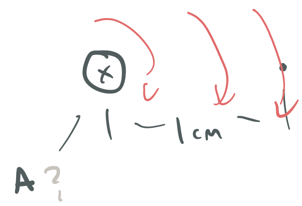
$$= 4000\ J/m^3$$

for 1kg of iron $= 7860\ kg/m^3$

so $4000\ \frac{J}{m^3} \cdot \frac{1}{7860}\frac{m^3}{kg} = 0.48\ J/kg$

at 60 Hz $\left(\frac{1}{s}\right)\times$ 60 W for 1 kg

13.6)



we need to apply a field ≈ the coercivity of gamma ferric oxide = 300 Oe

= 24e3 A/m

we can use biot-savart

orthogonal so cross product = I

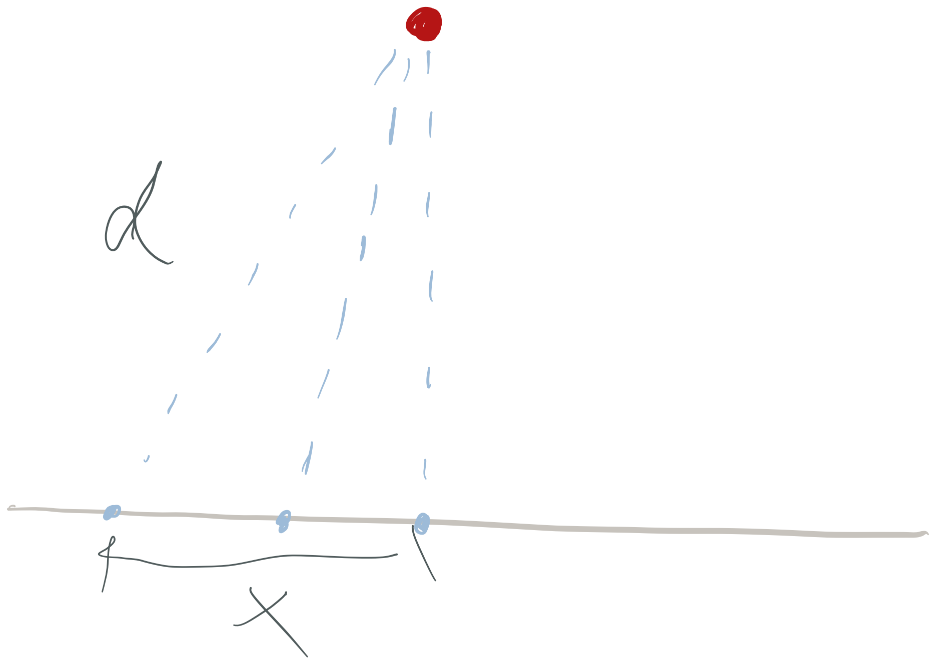
$$H(r) = \frac{1}{4\pi}\int_L \frac{I\,dl \times \hat{r}'}{|r'|^2} = \frac{1}{4\pi}\,\frac{I \times (1)}{(.01\,m)^2}$$

$$24e3\ \frac{A}{m} = \frac{1}{4\pi}\,\frac{I}{0.01^2}$$ solving for

$I = 30.144\ A$

$$= \frac{1}{4\pi}\int \frac{I\,dl \times \hat{r}'}{|r'|^2} = \frac{1}{4\pi}\,\frac{I\,l \times \hat{r}}{|r'|^2}$$

$$= \frac{I\,\|l\| \cdot \|\overset{1}{\hat{r}}\| \cdot \cos 90}{0.01^2}\left(\frac{1}{4\pi}\right)$$



$$H = \frac{I}{2\pi r} = \frac{I}{4\pi r^2}$$

2r