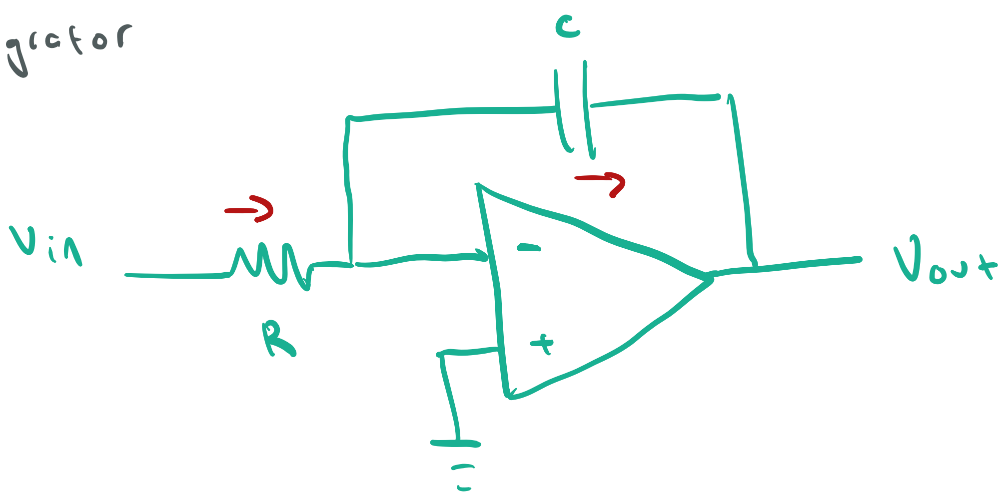
15.1) a)

(ideal) op amps: current into inputs is 0

$V_{out} = G \cdot \Delta input$

inputs are kept the same

Integrator



Using KCL (Kirchhoff's current law)

current through R:

$V = IR \qquad I = \frac{V_{in}}{R}$

current through cap

$I = C \frac{dV_o}{dt}$

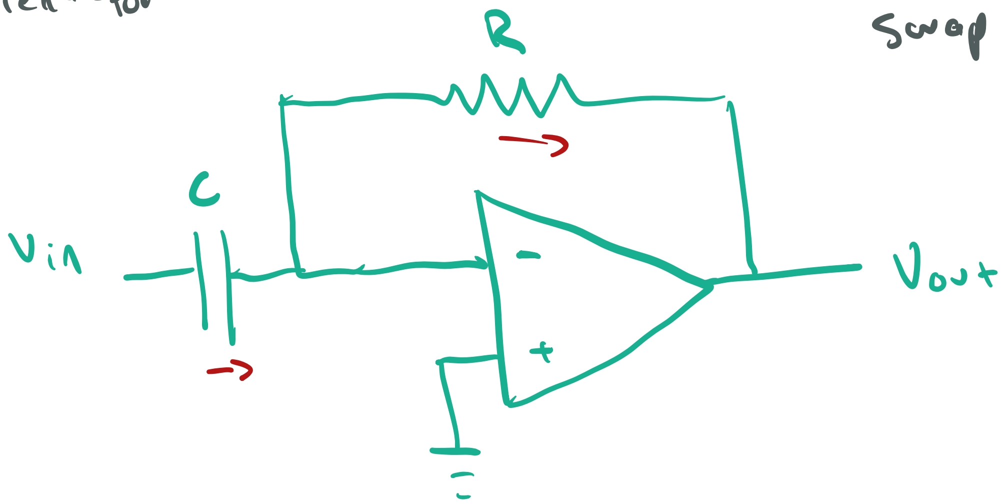
R + C are constants

$$\frac{V_{in}}{R} = C \frac{dV_o}{dt}$$

$$V_o = -\frac{1}{RC} \int V_i \, dt$$

Differentiator

Swap R + C

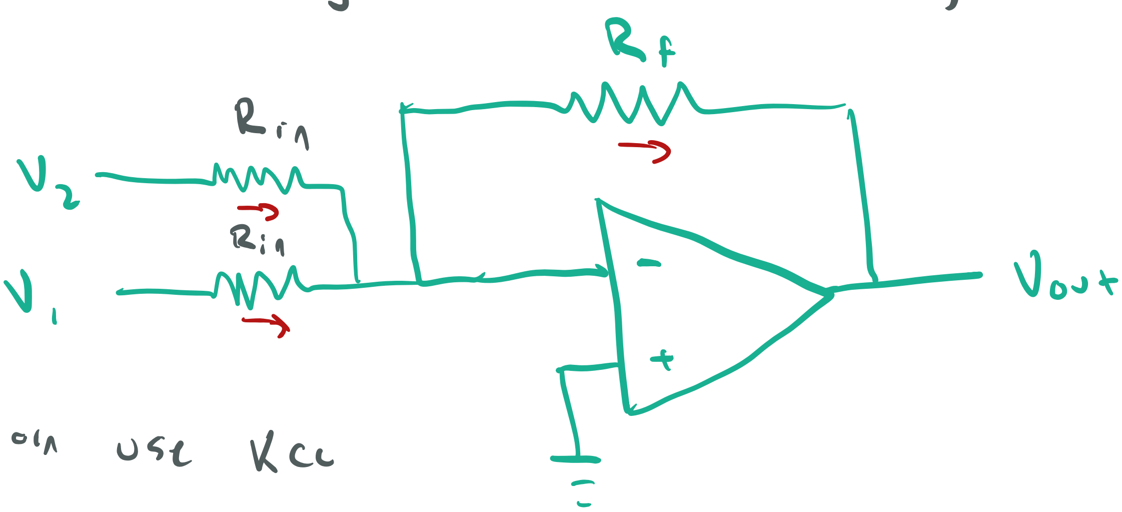


- otherwise opposite direction

now: $C\frac{dV_i}{dt} = \frac{-V_o}{R}$

$V_o = -RC\frac{dV_i}{dt}$

Summing (assuming all R_{in}s are the same)

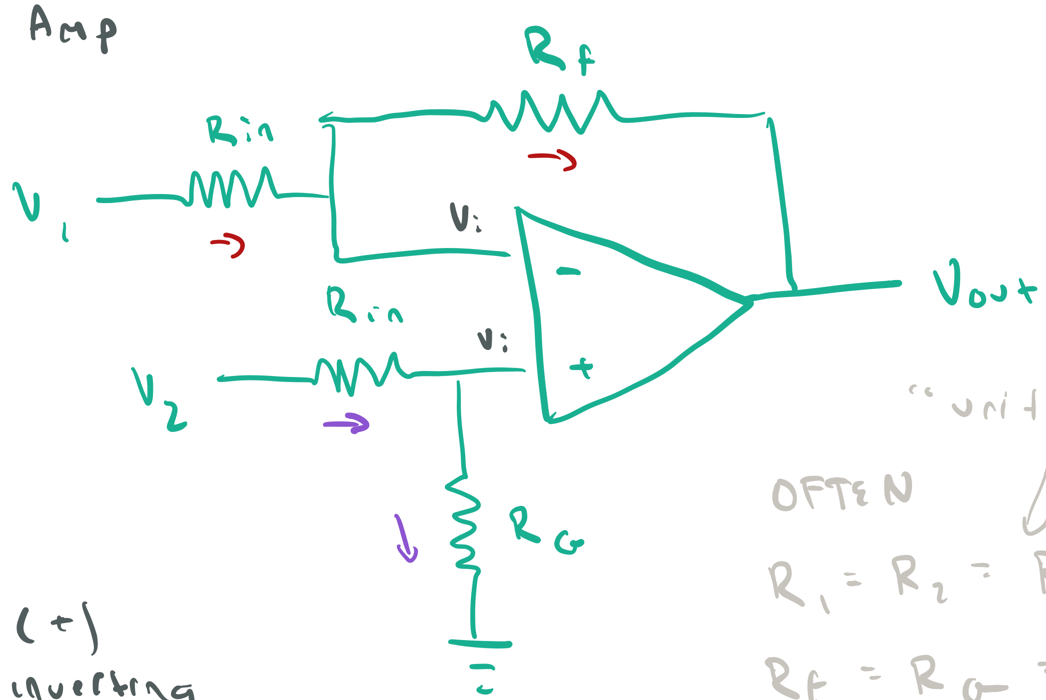


again use KCL

$\frac{\sum V_{in}}{R_{in}} = \frac{-V_o}{R_f}$

$V_o = -\frac{\sum V_{in}}{R_{in}}(R_f)$

Differential Amp



"unity gain"

OFTEN

$R_1 = R_2 = R_{in}$

$R_f = R_G =$

KCL to non-inverting (+)

$$\frac{V_2 - V_{in}}{R_{in}} = \frac{V_{in}}{R_G}$$

$$\frac{V_2}{R_{in}} = \frac{V_{in}}{R_G} + \frac{V_{in}}{R_{in}}$$

$$\frac{V_2}{R_{in}} = V_{in}\left(\frac{1}{R_G} + \frac{1}{R_{in}}\right)$$

KCL to inverting (−)

$$\frac{V_1 - V_{in}}{R_{in}} = \frac{V_{in} - V_0}{R_f}$$

write it all out

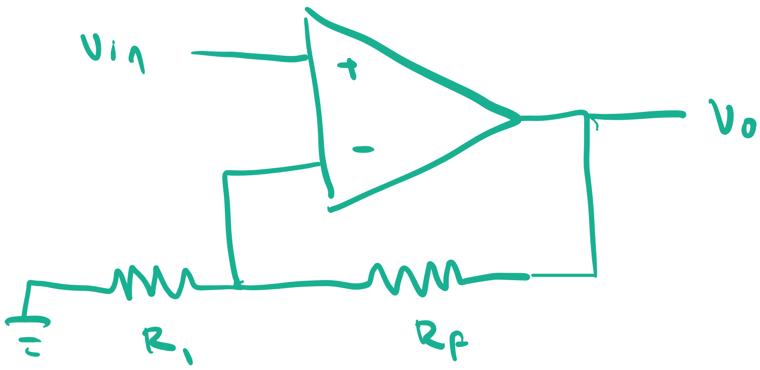
$$\frac{V_1}{R_{in}} - \frac{V_{in}}{R_{in}} = \frac{V_{in}}{R_f} - \frac{V_0}{R_f}$$

$R_f = R_G$

$$\frac{V_1}{R_{in}} + \frac{V_0}{R_f} = V_{in}\left(\frac{1}{R_{in}} + \frac{1}{R_f}\right) = \frac{V_2}{R_{in}}$$

$$V_0 = \left(\frac{V_2}{R_{in}} - \frac{V_1}{R_{in}}\right)(R_f) \quad \therefore \quad V_0 = \frac{R_f}{R_{in}}(V_2 - V_1)$$

15.1) b)



KCL:

$$\frac{V_o - V_{in}}{R_f} = \frac{V_{in}}{R_1} \qquad V_o = \frac{R_f}{R_1} V_{in} + V_{in}$$

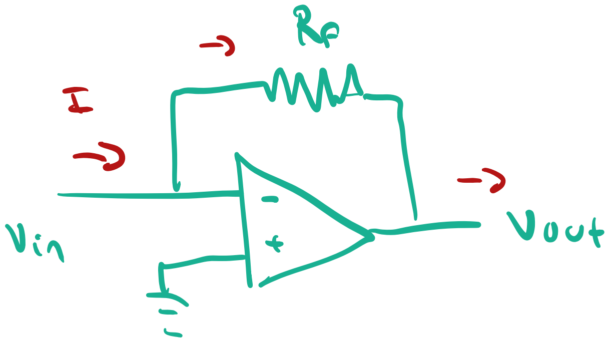
$$= V_{in}\left(\frac{R_f}{R_1} + 1\right)$$

one downside is that we can only amplify up because of the +1 (an inverting amp is $= -V_{in}\frac{R_f}{R_{in}}$

Also we have R_1 between Gnd + our inverting input so poorer common mode noise rejection

15.1) c)

Transimpedance Vout proportional to I_{in}

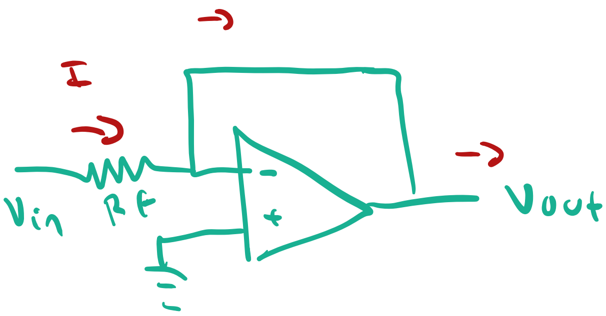


KCL

$$I = \frac{V_o}{R_f}$$

so proportional to R_f

Transconductance I_{out} proportional to V_{in}



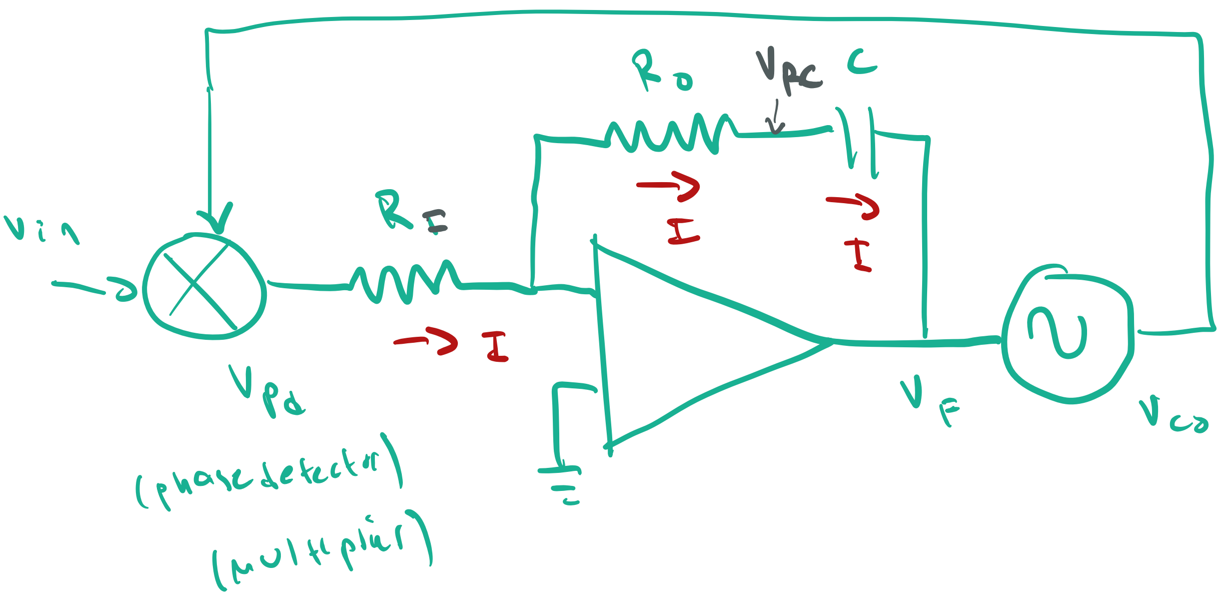
$$I_{out} = \frac{V_{in}}{R_f}$$

15.1) d) Lock in amps amplify weak + repetetive signals buried in noise

Input 1 = noisy signal

Input 2 = reference signal with same frequency as desired signal

Output represents amplitude of desired signal + phase relationship to reference.



we are given

$$\frac{dV_F}{dt} = -\frac{R_0}{R_I} \cdot \frac{dV_{pd}}{dt} - \frac{V_{pd}}{R_I C}$$

using KCL : $I_{R_I} = I_{R_0} = I_C$

for a cap

$I = C \frac{dV}{dt}$

(1) $I = \frac{V_{pd}}{R_I}$ -or- (2) $I = \frac{-V_{RC}}{R_0}$ -or- (3) $I = C\left(\frac{dV_{RC}}{dt} - \frac{dV_F}{dt}\right)$

differentiate

$$\frac{dI}{dt} = \frac{dV_{pd}}{dt\ R_I} \qquad \frac{dI}{dt} = -\frac{dV_{RC}}{dt\ R_0}$$

plug these into (3)

$$\frac{dV_{RC}}{dt} = -\frac{dV_{pd}}{dt} \cdot \frac{R_0}{R_I}$$

so: $$I = C\left(-\frac{dV_{pd}}{dt}\frac{R_0}{R_I} - \frac{dV_F}{dt}\right) = \frac{V_{pd}}{R_I}$$

now we solve for $\frac{dV_F}{dt}$

$$\frac{dV_F}{dt} = -\frac{V_{pd}}{R_I \cdot C} - \frac{dV_{pd}}{dt} \cdot \frac{R_0}{R_I}$$

(sign is just convention)