1st HERE'S physical intuition for the variables

n = # of steps / observations (# of head flips)

p = prob of outcome (50% heads)

$N = np$ (no physical intuition)?

x = # of favorable outcomes (# of heads landing)

eg. $n = 10$ for 10 head flips, prob. of $x = 2$ 2 being heads

$$P_n(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

makes sense if you plug in 1

σ^2 = variance

σ = the population standard deviation

$\langle x \rangle$ (also written) $E(x)$ is the expected value or "average"

3.1 (a) POISSON DISTRIBUTION

$$(3.16)\quad p(x) = \frac{e^{-\nu}\,\nu^{x}}{x!}$$

$$p_n(x) = \binom{n}{x} p^{x} (1-p)^{n-x} \qquad \text{where } \binom{n}{x} = \frac{n!}{(n-x)!\,x!}$$

sub

$$= \frac{n!}{(n-x)!\,x!}\, p^{x} (1-p)^{n-x}$$

take ln of of both sides

using ln quotient rule $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$$\ln(p_n(x)) = \ln(n!) - \ln[(n-x)!] - \ln x! + \ln p^{x} + \ln(1-p)^{n-x}$$

Stirling's approx for large n: 1) $n! \approx \sqrt{2\pi}\, n^{n+1/2} e^{-n}$

sub 1) & 2)

2) $\ln(n!) \approx n\ln(n) - n$
$\approx n\ln(n)$

3) $\ln(1+x) \simeq x$

$$\ln(p_n(x)) = n\cdot\ln(n) - (\cancel{n}-x)\cdot\ln(n-x) - \ln(x!) + x\ln(p)$$
$$+ (n-x)\cdot\ln(1-p)$$

$\ln(x^{y}) = y\ln x$
log of power

TALK TO ERIK

$n \cdot \ln(n) \approx n \cdot \ln(n-x)$ for large n & small x

$$\ln(p_n(x)) = x \cdot \ln(n-x) - \ln(x!) + x\ln(p) + (n-x)\ln(1-p)$$

for large n small p $(n-x)\ln(1-p) = -np$

$$\ln(p_n(x)) = x\ln(n-x) - \ln x! + x\ln p - np$$

now recombine everything

$N = np$ given to be avg # of events

$$p_n(x) = \frac{n^x p^x e^{-np}}{x!} = \frac{(np)^x e^{-np}}{x!} = \frac{N^x e^{-N}}{x!}$$

3.1 (b)

PROVE: $\langle x(x-1)(x-2)\cdots(x-m+1)\rangle = N^m$

Start of chapter we are given:

$$\langle f(x)\rangle = \int f(x)p(x)\,dx$$

eq. 3.16

$$\langle x(x-1)(x-2)\cdots(x-m+1)\rangle$$

THEREFORE THE FACTORIAL MOMENTS OF THE POISSON DIST

are given by:

$$\langle x(x-1)(x-2)\cdots(x-M+1)\rangle$$

$$= \sum_{x=0}^{\infty} \frac{e^{-N} N^x}{x!} x(x-1)(x-2)\cdots(x-M+1)$$

A

shown that this is $= 1$, so we can multiply it in

because $\frac{x(x-1)(x-M+1)}{x!} = \frac{1}{(x-M)!}$

$$\therefore \sum_{x=0}^{\infty} \frac{e^{-N} N^x}{(x-M)!}$$

using $N^x = N^M \cdot N^{(x-M)}$

$$\sum_{x=0}^{\infty} \frac{e^{-N} N^{x-M}}{(x-M)!} \cdot N^M = N^M$$

if we say $x = x - M$

$\therefore$ this whole thing is $= 1$

3.1c derive $\frac{\sigma}{\langle x \rangle} = \frac{1}{\sqrt{N}}$

1) $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 \longrightarrow$ we want get this into a form with just N

2) $\langle x \rangle = N = np$

so using 1) + 2) $\sigma^2 = \langle x^2 \rangle - N^2$

Now lets convert

3) $\langle x^2 \rangle = \langle x(x-1) \rangle + \langle x \rangle$ just algebra (pull out the $\langle x \rangle$)

convert this now

just N

using result from 3.1b

$\langle x(x-1) \rangle = N^2$ because... do it out in 3.1b + watch Amira's video

$\therefore \sigma^2 = N^2 + N - N^2$

$\sigma^2 = N \Rightarrow \sigma = \sqrt{N}$

so $\frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$

(reminder that

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$$
)

3.2 we can model photon creation as a poisson process. (random + independent)

we know std dev: $\sigma = \sqrt{N}$

where N = photons detected/second

$\therefore$ to measure to within 1%

we must ensure

(10^{-2})

1% $\sigma \leq 0.01 N$ $\therefore$ we need $\sqrt{N} \leq 0.01 N$

or $N \geq 10^4$

1ppm $\sigma \leq 10^{-6} N$ $\therefore$ $N \geq 10^{12}$

for wattage:

$\lambda_{visible} = 500 \cdot 10^{-9}$ m

$$E = \frac{hc}{\lambda} = \frac{6.626 \cdot 10^{-34} \text{ Js} \cdot 3 \cdot 10^{8} \text{ m/s}}{500 \cdot 10^{-9} \text{ m}}$$

$= 3.8 \cdot 10^{-19}$ J, for W just multply by rate/s

3.3a solve for v_{source}

amp bandwidth of $20\ kHz = f$

$Z = 10\ k\Omega$

$T = 300K$

SNR of 20 db

Johnson Noise is thermal gradients causing charge to jiggle around

$$\overline{V_n^2} = 4 K_B\ T \cdot R \cdot f$$

$$= 3.3 \cdot 10^{-12}\ V^2$$

for power $\quad dB = 10 \log_{10}\left(\frac{V_{sig}^2}{V_n^2}\right)$

$$20 = 10 \log_{10}\left(\frac{V_{sig}^2}{3.3 \cdot 10^{-12}}\right)$$

we know

$$2 = \log_{10}(100) \therefore 100 = \frac{V_{sig}^2}{3.3 \cdot 10^{-12}} \quad V_{sig}^2 = 3.3 \cdot 10^{10}\ V$$

3.3 b

from textbook
Equipartition Theorem:

energy in a cap: $E_0 = \frac{CV^2}{2}$

also each "thermalized kinetic DOF" has E_0 of

$$E_0 = 1/2\, K_B \cdot T$$

solving for C

$$\frac{CV^2}{2} = \frac{K_B T}{2} \qquad C = \frac{K_B T}{V_{noise}^2}$$

$$C = \frac{1{,}38 \cdot 10^{-23}\ J/K \cdot 300\ K}{3{,}3 \cdot 10^{-12}\ V^2}$$

$$C = 1{,}25 \cdot 10^{-9}\ F$$

3.3c

RMS shot noise from text

$$\langle I^2 \rangle_{noise} = 2q \langle I \rangle \Delta f$$

$q = 1.602e^{-19}$ C (electron charge)

If we want 1% noise relative to current:

$$I_{noise} = 0.01 I$$

so

$$\frac{\langle I^2_{noise} \rangle = 2q \cdot I \cdot \Delta f}{I} = \frac{(0.01 I)^2}{I}$$

$$I = \frac{2q \Delta f}{(0.01)^2} = \frac{2 \cdot 1.6e^{-19} \cdot 20\text{ kHz}}{0.01^2}$$

$$I = 6.4 \cdot 10^{-11}\text{ A}$$