8.2

magnitude of poynting

$$\langle |P| \rangle = \frac{1000\ W}{\text{area of sphere @ that radius}} = \frac{1000\ W}{4\pi (1000\ m)^2}$$

$$= 8 \cdot 10^{-5}\ W/m^2$$

$$\|\vec{P}\| = \|\vec{E} \times \vec{H}\|$$

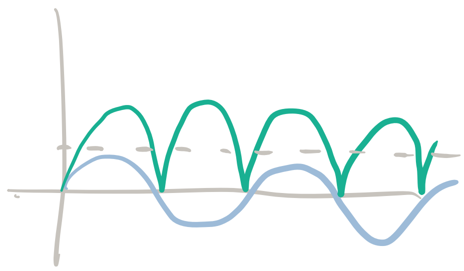
where $\vec{E}$ + $\vec{B}$ are perp (free space) + $\vec{P}$ points radially out from the point source

To write in terms of $\vec{E}$ only

$$H = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E$$

$$P = \sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_{max}^2 \left(\frac{1}{2}\right)$$

for time average of a sin wave squared

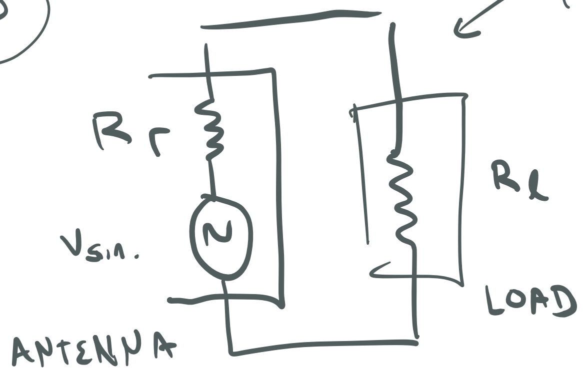



plugging in + solving for $E_{max}$

$$E_{max} = \left(\sqrt{\frac{\varepsilon_0}{\mu_0}} \cdot \frac{1}{2\langle P \rangle}\right)^{1/2}$$

$$= 0.24\ V/m$$

8.3




we know from lecture that we want an impedance match to minimize reflection

we want to solve for power

$$P = I^2 R_\ell$$

sub in

From Ohm's law

$$V = I(R_r + R_\ell)$$

$$I = V / R_r + R_\ell$$

$$P = \frac{V^2}{(R_r + R_\ell)^2} R_\ell$$

classic peak is at 0 derivative problem

$$P = \frac{V^2 R_\ell}{R_r^2 + R_R R_\ell + R_\ell^2} = V^2 R_\ell \left(R_r^2 + R_R R_\ell + R_\ell^2\right)^{-1}$$

$$\frac{dP}{dR_\ell} = 0 = \underbrace{V^2 R_\ell}_{a} \underbrace{\left(R_r^2 + R_\ell R_r + R_\ell\right)^{-1}}_{b}$$

$$= da \cdot b + a \cdot db$$

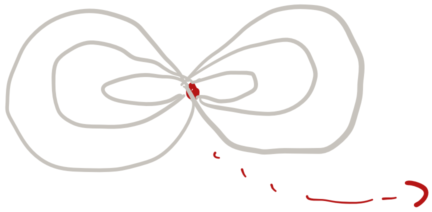
$$0 = \frac{\cancel{V^2}}{\cancel{(R_\ell + R_r)^2}} - \frac{2 \cancel{V^2} R_\ell}{(R_\ell + R_r)^3}$$

$$\frac{2R_\ell}{R_\ell + R_r} = 1 \Rightarrow 2R_\ell = R_\ell + R_r$$

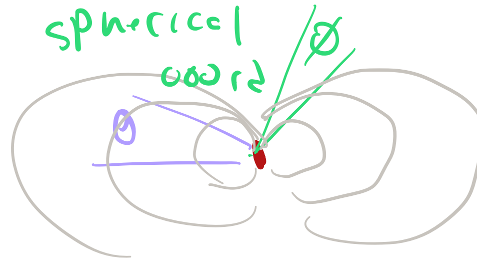
$$R_\ell = R_r$$

nice!

8.4



gain ∝ area?




$\boxed{I\,d}$ height but infinitesimal

$\phi$ is meaningless

## GAIN

We are given:

$$\langle|P|\rangle = \frac{I_0^2 k^2 d^2}{32\pi^2 r^2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\sin^2\theta$$

(mag. of poynting)

where

$I_0 = |I_{max}|$ ?

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \quad \frac{\text{frequency}}{\text{speed of light}}$$

$$w = \frac{I_0^2\pi}{3}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{d}{\lambda}\right)^2$$

(total energy radiated)

Therefore gain is:

gain is defined as $\frac{\text{Poynting max}}{\text{power}/4\pi}$

$P_{max}$ is at $\theta = 90$ so $\sin 90 = 1$

+ $r=1$ to simplify

+ no phi dependence

$$G = \frac{\langle|P(r=1,\theta_{max},\phi_{max})|\rangle}{w/4\pi}$$

$$= \cancel{4}\pi\,\frac{\cancel{I_0^2}\,k^2\,\cancel{d^2}}{8\,\cancel{32}\,\pi^2 r^2}\sqrt{\frac{\cancel{\mu_0}}{\cancel{\varepsilon_0}}}\sin^2\theta\cdot\frac{3}{\cancel{I_0^2}\cancel{\pi}}\sqrt{\frac{\cancel{\varepsilon_0}}{\cancel{\mu_0}}}\left(\frac{\lambda}{\cancel{d}}\right)^2$$

$\frac{2\pi}{\lambda}$

$$= \frac{3k^2\lambda^2\cdot 4}{8\pi^2 r^2} = \frac{12}{8r^2}\ @\ r=1 = \frac{3}{2}$$

AREA

we know $P_{max} = V^2 / 8R_{rad}$

& $P_{max} = A \cdot \langle |\bar{P}| \rangle$

$$= \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E^2_{max} \cdot A$$

so $A = \frac{2V^2}{8R_{rad} E^2_{max}} \sqrt{\frac{\mu_0}{\varepsilon_0}}$

we are given $R_{rad} = \frac{2\pi}{3}\sqrt{\frac{\mu_0}{\varepsilon_0}}\left(\frac{d}{\lambda}\right)^2$

$$= \frac{3}{8\pi} \frac{V^2}{E^2_{max}} \left(\frac{\lambda}{d}\right)^2$$

for an infinitesmal length d, E will be constant (max) across it. so we know from last week $V = -\int \vec{E} \cdot dl$

$\therefore V = E_{max} \cdot d$

$\therefore \Rightarrow \frac{3}{8\pi}\lambda^2$

RATIO

$$\frac{A}{G} = \frac{3}{8\pi}\lambda^2 \left(\frac{3}{2}\right)^{-1}$$

$$= \lambda^2 / 4\pi$$

8.1