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PSET10

13.1

a.

From the book we have:

$$\chi_m = -\frac{\mu_0 q^2 Z r^2}{4m_e V}$$

Let's do this for water... I think. We have 10 electrons, $r=0.19\text{nm}$, and $V=3 \times 10^{-29}$.

$$\chi_m = -4\pi * 10^{-7} * (1.6 * 10^{-19})^2 * 10 * \frac{(0.19 * 10^{-9})^2}{4 * 9.1 * 10^{-31} * 3 * 10^{-29}}$$
$$\chi_m = -1 * 10^{-4}$$

This is an order of magnitude off from the real value, which is $\approx -10^{-5}$, So there's probably a typing error or I don't understand what's going on, which is also quite possible!

b.

We have:

$$F = -V\mu_0\chi_m H \frac{dH}{dz}$$

We will do this for frog \approx water.

The frog's weight will be 22.7g, or 0.0227kg. We'll say $V = 2.27 \times 10^{-5}$ then.

$$0.0227 * 10 = -2.27 * 10^{-5} * 4 * \pi * 10^{-7} * -10^{-5} * H \frac{dH}{dz}$$

Now let's estimate $\frac{dH}{dz}$. If the gradient goes to zero outside the frog, then we can approximately say $\frac{dH}{dz} = \frac{H}{r} = H$, so $H = H/0.017$.

$$H = 3.7 * 10^6$$
$$B = \mu_0 H = 4.6[T]$$

13.2

$$\vec{B} = \frac{\mu_0}{4\pi} \left[\frac{3\hat{x}(\hat{x} \cdot \vec{m}) - \vec{m}}{|\vec{x}|^3} \right]$$

We are given:

$$U = -\vec{m} \cdot \vec{B}$$

This will be max/min when $\cos \theta = 0$, or when m and B are antiparallel/parallel.

$$U = -|\vec{m}| \frac{\mu_0}{4\pi} \left[\frac{4|\vec{m}|}{|\vec{x}|^3} \right] = \frac{m^2 \mu_0}{4\pi x^3}$$

For this, $m = -9.28 \times 10^{-24}$ and $x = 10^{-10} \text{m}$.

Then,

$$\begin{aligned} U &= (-9.28 \times 10^{-24})^2 * \frac{4 * \pi * 10^{-7}}{4 * \pi * 10^{-30}} \\ &= 8.6 * 10^{-24} [J] \end{aligned}$$

Electrostatic energy is given by:

$$\begin{aligned} U &= qV = \frac{q^2}{4\pi\epsilon_0 r} \\ &= \frac{(1.6 * 10^{-19})^2}{4\pi(8.85 * 10^{-12}) * 10^{-10}} \\ &= 2.3 * 10^{-18} [J] \end{aligned}$$

That's six orders of magnitude larger.

13.3

a.

$$U = \frac{1}{2} (\vec{E} \cdot \vec{D} + \vec{B} \cdot \vec{H})$$

Or, for no E fields,

$$\begin{aligned} U &= \frac{1}{2} \int \vec{B} \cdot \vec{H} dV \\ &= \frac{1}{2} V \frac{1}{\mu} B^2 \end{aligned}$$

For a ferromagnet, μ_r is >1000 s, so U is smaller if the field goes through the material as opposed to whatever non-ferromagnetic medium it's in.

b.

$$\begin{aligned} U &= \frac{1}{2} \int \vec{B} \cdot \vec{H} dV \\ \vec{H} &= \frac{\vec{B}}{\mu_0} - \vec{M} \end{aligned}$$

$$U = \frac{1}{2} \int \vec{B} \cdot \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) dV$$

This will be minimized when $\vec{B} \cdot \vec{M}$ is maximized, or when they are aligned. What this means at a physical level is that the magnetic field inside the magnet is aligned with the magnetization of the other magnet, i.e. that the opposite poles attract.

13.4

Google says that iron has 2 electrons in its outermost shell. The density is 7.874 g/cm³ near room temperature, so we'll just round to 8. Its atomic mass is 55.845 u. We have

$\mu_B = 9.27 * 10^{-24} \left[\frac{J}{T} \right]$ for the conduction electrons in metal

$$\frac{9.27 * 10^{-24} \left[\frac{J}{T} \right]}{[electron]} * \frac{2 [electron]}{[atom]} * \frac{(6.022 * 10^{23}) [atom]}{55.85[g]} * \frac{8[g]}{cm^3} = 1.6 \left[\frac{J}{Tcm^3} \right]$$

Or:

$$= 1.6 * 10^6 \left[\frac{A}{m} \right]$$

Which are the units of magnetization, which I'm not 100 sure checks out from those units... but... we'll take it, lmao.

13.5

I'm skipping this problem for now. (forever?)

13.6

The B-field of a straight wire is given by:

$$B = \frac{\mu_0 I}{2\pi r}$$

We are given that the coercivity of γ -Fe₂O₃ is 300Oe. Seeing as these units are Oe, let's swap the above equation to:

$$H = \frac{I}{2\pi r}$$

Let's convert back into MKS for my poor little brain's sake:

$$300[Oe] = 300 * \frac{10^3}{4\pi} \left[\frac{A}{m} \right]$$

So:

$$300 * \frac{10^3}{4\pi} \left[\frac{A}{m} \right] = \frac{I [A]}{2 * \pi * 10^{-2} [m]}$$
$$I = 1500 [A]$$

Yeet, that's huge.