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PSET 11

14.1

Equation 14.6 is:

$$E \approx 2E_F - 2E_c e^{-\frac{2}{N_F V}}$$

Let's do derivative 1:

$$\frac{dE}{dV} = -\frac{4E_c e^{-\frac{2}{N_F V}}}{NV^2}$$

And guess what! This is not defined for V=0. And we'll just leave this at that!

14.2

$$IV = mgv$$

The inverse AC Josephson effect is:

$$V = n\frac{h}{2e}f$$

The quantum hall effect:

$$R_H = \frac{1}{i}\frac{h}{e^2}$$

We have Ohm's law:

$$V = IR$$

So, we have:

$$\frac{V^2}{R} = \frac{\left(n\frac{h}{2e}f\right)^2}{\frac{1}{i}\frac{h}{e^2}} = mgv$$

$$mgv = \frac{in^2hf^2}{4}$$

14.3

One fluz quantum is given as:

$$\Phi_0 = 2.07 \times 10^{-7}[G \cdot cm^2]$$

For A=1cm^2:

$$B = 2.07 * 10^{-7}[G]$$

In SI units:

$$B = 2.07 \times 10^{-11}[T]$$

For an infinitely long straight wire:

$$B = \frac{\mu_0 I}{2\pi r}$$

So:

$$2.07 \times 10^{-11} = 4\pi * 10^{-7} * \frac{1}{2 * \pi * r}$$

$$r = 2 * \frac{10^{-7}}{2.07 * 10^{-11}}$$

$$r = 9.66 * 10^3$$

14.4

Skipping for now…

14.5

The chronometer has a relative error of 10^-5.

One month = 2.628*10^6s.

So in one month, we will be off by a maximum of ~26 seconds.

The surface of the Earth moves at ~460m/s at the equator, so if we can precisely locate our angle change relative to the heavens, we could be off by ~12km.

The cesium beam is 10^-12 relative uncertainty, or 2.6*10^-6 seconds, or 0.0012m.

14.6

GPS satellites orbit at 20,180km.

a.

The gravitational force on the satellite is given by:

$$F = \frac{Gm_1 m_2}{r^2}$$

The gravitational force is acting as a centripetal force, i.e.:

$$\frac{Gm_1 m_2}{r^2} = \frac{m_1 v^2}{r}$$

Solving for v:

$$v = \sqrt{\frac{Gm_2}{r}}$$

Evaluating:

$$v = \sqrt{\frac{6.67 * 10^{-11} * 5.97 * 10^{24}}{6.37 * 10^6 + 20180 * 10^3}}$$

$$v = 3.87 * 10^3 \left[\frac{m}{s}\right]$$

b.

We make circles of circumference:

$$2 * \pi * (6.37 * 10^6 + 20180 * 10^3) = 1.67 * 10^8 [m]$$

The period is then:

$$T = 1.67 * 10^8 / (3.87 * 10^3)$$

$$T = 4.3 * 10^4 [s]$$

Which in hours is 11.9.

c.

Special relativity gives us:

$$t_e = \frac{t_s}{\sqrt{1 - \frac{v^2}{c^2}}}$$

So:

$$t_e - t_s = t_e \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right)$$

$$= 4.3 * 10^4 * \left(1 - \sqrt{1 - \frac{(3.87 * 10^3)^2}{(3 * 10^8)^2}}\right)$$

$$= 3.5778 * 10^{-6}$$

So it goes a tiny bit slower.

d.

For this, we have:

$$t_e = \frac{\left(1 - \frac{GM}{rc^2}\right)}{1 - \frac{GM}{r'c^2}} t_s$$

So:

$$t_e - t_s = t_e \left(1 - \frac{\left(1 - \frac{GM}{r'c^2}\right)}{1 - \frac{GM}{rc^2}}\right)$$

$$= 4.3 * 10^4 * \left(1 - \frac{\left(1 - 6.67 * 10^{-11} * 5.98 * \frac{10^{24}}{(6.37 * 10^6 + 20180 * 10^3) * (3 * 10^8)^2}\right)}{1 - 6.67 * 10^{-11} * 5.98 * \frac{10^{24}}{(6.37 * 10^6) * (3 * 10^8)^2}}\right)$$

$$= -2.27 * 10^{-5}$$

So from this, we get that the satellite is has a faster time. I'm not sure if the special and general effects just add, but if they do, we get that overall the satellite goes speedier.