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PSET 5

7.1

There are basically two types of noise we're trying to shield with the twisted pair + grounded shield: inductive and capacitive coupling. If we visualize the twisted pair (one line takes signal out, the other back in), and move it in a B-field, we expect that the induced currents will cancel each other out. The grounded shield, uh, shields out E-fields (any induced potentials will exist in the shield, by grounding it, we're able to cancel those out).

7.2

We are given:

$$\delta = \frac{1}{\sqrt{\pi \nu \mu \sigma}}$$

For the problem, we have: $\sigma = 4\left[\frac{s}{m}\right], \nu = 10^4[Hz], \mu \approx 1.25 * 10^{-6}$
So:

$$\delta = \frac{1}{\sqrt{3.14 * 10^4 * 1.25 * 10^{-6} * 4}} = 2.5[m]$$

7.3
The Poynting vector is:

$$\vec{P} = \vec{E} \times \vec{H}$$

We are to integrate this over a coaxial cable to find the power flowing through a cross-sectional slice of it:

$$\int_0^{2\pi} \int_{r_0}^{r_1} \left(\vec{E} \times \vec{H}\right) r dr d\theta$$

We are given that $\left|\vec{H}\right| = \frac{I}{2\pi r}$ and $\left|\vec{E}\right| = \frac{Q}{2\pi \epsilon r}$ for this geometry. $\vec{H}$ will point in the $\hat{\theta}$ direction and $\vec{E}$ will point in the $\hat{r}$ direction, so we can surmise that that $\vec{P}$ points in the let's call it $\hat{z}$ direction (or out of the coax).

$$\int_0^{2\pi} \int_{r_0}^{r_1} \frac{I}{2\pi r} \frac{Q}{2\pi \epsilon r} r dr d\theta$$

$$= \frac{QI}{4\pi^2 \epsilon} \theta \int_{r_0}^{r_1} \frac{1}{r} dr \Big|_0^{2\pi}$$

$$= \frac{QI}{2\pi\epsilon} \ln\left(\frac{r_1}{r_0}\right)$$

We are given that

$$V = \frac{Q}{2\pi\epsilon} \ln\left(\frac{r_1}{r_0}\right)$$

For this geometry, so the power flowing through this cross sectional region is

$$= IV$$

Which we could expect.

7.4

We find the characteristic impedance of two parallel strips with width w and distance d.

$$Z = \sqrt{\frac{L}{C}}$$

Let's find C first. This geometry is a parallel plate capacitor, so:

$$C = \frac{\epsilon w l}{d}$$

For some sub-length l.

Switching to Farads per meter:

$$C = \frac{\epsilon w}{d}$$

Inductance is given by:

$$L = \frac{\Phi}{I} = \frac{1}{I} \int \vec{B} * d\vec{A}$$

Let's find the B field from this flat sheet. Our constitutive equation:

$$\vec{B} = \mu \vec{H}$$

Via Stokes':

$$\oint \vec{H} d\vec{l} = \iint \left(\vec{J} + \frac{\partial D}{\partial t}\right) d\vec{A}$$

In our case, we can draw a little loop around one of our sheets (looking at it in cross section). (This is not unlike the solenoid question from last week's homework).

$$\vec{H} w = I$$

So:

$$\vec{B} = \frac{\mu I}{w}$$

The field is going to be running like through the plates, orthogonal to their length.
This gives us:

$$L = \frac{\mu}{w} d * l$$

Switch to per length:

$$L = \frac{\mu}{w} d$$

Then:

$$Z = \sqrt{\frac{\frac{\mu}{w} d}{\frac{\epsilon w}{d}}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}}$$

And

$$v = \frac{1}{\sqrt{LC}}$$

So:

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

7.5

a.

The inductance per length is:

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{r_0}{r_1}\right)$$

The capacitance per length is:

$$C = \frac{2\pi\epsilon}{\ln\left(\frac{r_0}{r_1}\right)}$$

$$Z = \sqrt{\frac{L}{C}} = \ln\left(\frac{r_0}{r_1}\right)\sqrt{\frac{\mu_0}{4 * \pi^2 * \epsilon}}$$

With the parameters:

$$r_1 = 0.406mm, r_0 = 1.48mm, \epsilon = 2.26$$

$$Z = 1.29 * \sqrt{\frac{10^{-7}}{\pi * \epsilon * \epsilon_0}} \approx 0.59 * 10^2$$

We know that this should be about 50Ω, so I will just round in the other direction :p

b.

$$v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$v = \frac{1}{\sqrt{4\pi * 10^{-7} * 2.26 * 8.85 * 10^{-12}}} \approx 2 * 10^8 \left[\frac{m}{s}\right]$$

c.

We will do this as $v * t_{clock} = d_{max}$

$$2 * 10^8 * 10^{-9} = 0.2[m]$$

d.

30 mils is 0.000762m.

$$50 = \ln\left(\frac{762}{r_i}\right)\sqrt{\frac{\mu_0}{\epsilon\epsilon_0}}$$
$$1.25 = \ln\left(\frac{762}{r_i}\right)$$
$$3.5 \approx \frac{0.762[mm]}{r_i}$$
$$r_i = 0.22[mm]$$

e.

$$v = \lambda * f$$
$$2 * 10^8 = 3 * 10^{-3} * f$$
$$f = 6.6 * 10^{10} = 66[GHz]$$

7.6

a.

Propagation delay: 4.6ns/m
Impedance: 100 ohms

Physical length of minimum size 64 byte frame. In bits, this is 512 bits.

Our speed is 1/propagation delay, i.e., 0.22*10^9 m/s.

We'll say our ethernet is Gigabit/s, because that is the top speed the internet says Cat6 can handle.

$$2.2 * 10^8 \left[\frac{m}{s}\right] * \frac{1}{10^9}\left[\frac{s}{bit}\right] * 512 \approx 112\ [m]$$

For the 64 byte frame.

b.

Resistors connected in parallel do the 1/over adding; impedances do the same.

So:

$$\frac{1}{Z_T} = \frac{1}{Z_{cat6}} + \frac{1}{Z_{cat6}} = \frac{1}{50}$$
$$Z_T = 50\Omega$$

The reflection coefficient is:

$$R = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50}{150} = \frac{1}{3}$$