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PIT 2022

PSET 8

11.1
a.

We are to derive:

$$\frac{\hbar^2}{2m} iq\left(A - B - Ae^{i(q-k)\Delta} + Be^{-i(q+k)\Delta}\right) = V_0(A+B)$$

From:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V\right]\psi = E\psi$$

$$\left[E + \frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right]\psi(x) = \sum_{n=-\infty}^{\infty} V_0 \delta(x - n\Delta)\psi(x)$$

Which is honestly very yikes! The problem statement is so short, and yet. And yet.

We integrate from $x = \epsilon$ to $-\epsilon$.

[] to do!

b.

11.2

This question is asking about the expected occupancy of a state at the conduction band edge for 3 materials. This means we're asking what's the expected amount of electrons we'll see in the conduction band, which is a jump of E_{gap} from the valence band. Since electrons are fermions, we have:

$$\langle N_S \rangle = \frac{1}{e^{\beta(E_s - \mu)} + 1}$$

$$\mu = \frac{\partial F}{\partial N} = E_F$$

$$\beta = \frac{1}{kT}$$

$kT = 0.026\ [eV]$ at room temperature.

The gap energies E_{gap} are:

Ge -> 0.67 eV
Si -> 1.11 eV
Diamond -> 5 eV

We're given that for intrinsic semi-conductors, which these all are, that $E_F = \frac{1}{2}E_{gap}$. So:

$$\langle N_S \rangle = \frac{1}{e^{\frac{1}{kT}\left(\frac{1}{2}E_{gap}\right)} + 1}$$

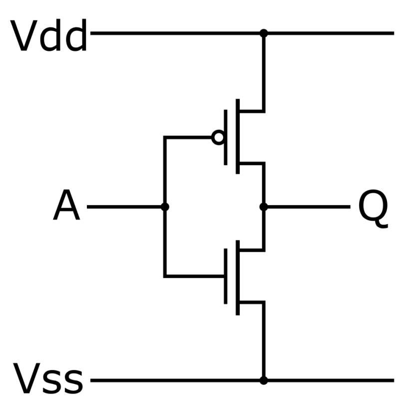
Plug and chugging, thnx Python:
Ge: `2.5367725832536544e-06`
Si: `5.363930900489389e-10`
Diamond: `1.741466809206832e-42`

11.3

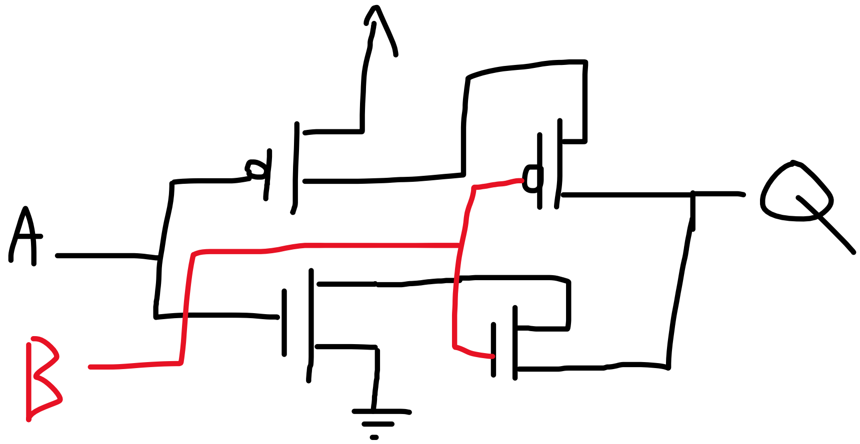
Skipping lolololololol

11.4

I do not know what a tri-state CMOS inverter is; I barely know what CMOS is. Don't tell anyone.



This is a plain inverter, the text says to add a control input to it that can force the output to a disconnected state. So I guess this is a two inputs to three outputs kind of thing, where our outputs are HIGH, LOW, and DISCONNECT. Lol.



I think this works. If we have HIGH on A and HIGH on B then we should have LOW on Q right. And vice-versa. But if we do LOW on A and HIGH on B we have a disconnect; same goes for the reverse. I don't know???

11.5

a.

Energy in a capacitor is given by:

$$U = \frac{1}{2}CV^2$$

For C=1fF and V=1.8V,

$$U = 1.62\ [fJ]$$

b.

The wire has resistance R. The current flowing to the capacitor at a time t is given by:

$$I = \frac{V}{R}e^{-\frac{t}{RC}}$$

The power is given by:

$$P = I^2R$$

$$P = \frac{V^2}{R}e^{-\frac{2t}{RC}}$$

The energy is given by the integral over time:

$$E = \int_0^\infty \frac{V^2}{R}e^{-\frac{2t}{RC}}dt$$

$$= -\frac{1}{2}V^2C\left(e^{-\frac{\infty}{RC}} - e^{-\frac{0}{RC}}\right)$$

$$= \frac{1}{2}CV^2$$
$$= 1.62[fJ]$$

c.

For total charge Q moved in time tau, the average current is given by:

$$I = \frac{Q}{\tau} = \frac{CV}{\tau}$$

From Ohm's law:

$$V_R = \frac{CVR}{\tau}$$

So we can get average power:

$$\frac{V^2}{R} = \frac{C^2V^2R}{\tau^2}$$

If we integrate the average power we get:

$$E = \int_0^{\tau} \frac{C^2V^2R}{\tau^2} dt = \frac{C^2V^2R}{\tau}$$

d.

Each charge-discharge cycle results in a CV^2 loss, or 3.25fJ.

$$freq * 3.25 * 10^{-15} = 1[W]$$
$$freq \approx 3 * 10^{14}\ [Hz]$$

e.

So we dissipate 3.25*10^-15 Joules per transistor, or 3.25*10^-6 Joules total for our Giga-transistor. For GHz clock, this comes out to:

$$3.26 * 10^3[W]$$

f.

$$C = \frac{Q}{V}$$
$$10^{-15} = \frac{Q}{1.8}$$
$$Q = 5.5 * 10^{-16}[Coulombs]$$

Electron charge: 1.6*10^-19 C

$= 3.4 * 10^3$ electrons.