

$$\boxed{2.1} \text{ a) } \frac{1 \text{ ymol}}{1} \cdot \frac{1 \text{ mol}}{10^{24} \text{ ymol}} \cdot \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} = \boxed{0.602 \text{ atoms}}$$

$$\text{b) } \frac{1 \text{ nanocentury}}{1} \cdot \frac{1 \text{ century}}{10^9 \text{ nanocentury}} \cdot \frac{100 \text{ years}}{1 \text{ century}} \cdot \frac{365 \text{ days}}{1 \text{ year}} \cdot \frac{24 \text{ hours}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hour}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} = \boxed{0.302 \text{ seconds}}$$

↳ near constant?

$$\boxed{2.2} \text{ DVD capacity} = 4.7 \text{ GB}, \text{ thickness} = 1.2 \text{ mm (Wikipedia)}$$

$$\frac{1 \text{ EB}}{1} \cdot \frac{10^9 \text{ GB}}{1 \text{ EB}} \cdot \frac{1.2 \text{ mm}}{4.7 \text{ GB}} \cdot \frac{1 \text{ km}}{10^6 \text{ mm}} = \boxed{255 \text{ km}}$$

$$\text{Distance to space} \approx 100 \text{ km} \rightarrow \approx \boxed{2.5 \times \text{dist. to space}}$$

$$\boxed{2.3} \text{ Atoms in universe} = 10^{80}$$

$$10^{80} \text{ bit \# where all bits are 1} \rightarrow \boxed{2^{(10^{80})} - 1}$$

↳ simplify?

$$\boxed{2.4} \text{ Gravitational acc. at Earth surface} \rightarrow g = 9.81 \text{ m/s}^2$$

$$1 \text{ kg mass @ 1 m} \rightarrow g = \frac{GM}{r^2} = \frac{6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2} \cdot 1 \text{ kg}}{1 \text{ m}^2} = 6.67 \times 10^{-11} \text{ m/s}^2$$

$$\text{Ratio} = 20 \log_{10} \left(\frac{9.81}{6.67 \times 10^{-11}} \right) = \boxed{223 \text{ dB}}$$

$\boxed{2.5}$ a) Assume energy stored in 1 ton of N_2 bonds

$$\text{Energy in } \text{N}_2 \text{ bond} = 945 \text{ kJ/mol}$$

$$\text{Molar mass of } \text{N}_2 = 28 \text{ g/mol}$$

$$\frac{1 \text{ ton}}{1} \cdot \frac{1000 \text{ kg}}{1 \text{ ton}} \cdot \frac{1000 \text{ g}}{1 \text{ kg}} \cdot \frac{1 \text{ mol } \text{N}_2}{28 \text{ g}} \cdot \frac{945 \text{ kJ}}{1 \text{ mol } \text{N}_2} \cdot \frac{1 \text{ GJ}}{10^6 \text{ kJ}} = \boxed{33.8 \text{ GJ per ton}}$$

b) Nuclear fission of Uranium-235 releases 2-3 neutrons per atom; assume 2.5 on average

According to Wikipedia, nuclear energy binding a neutron to a Uranium-235 nucleus = 7.6 MeV

Nuclear energy released by a gram of U-235:

$$\frac{1 \text{ g U-235}}{1} \cdot \frac{1 \text{ mol}}{235 \text{ g}} \cdot \frac{6.02 \times 10^{23} \text{ atoms}}{1 \text{ mol}} \cdot \frac{2.5 \times 7.6 \text{ MeV}}{1 \text{ atom}} \cdot \frac{1.60 \times 10^{-19} \text{ MJ}}{1 \text{ MeV}}$$

$$= 7788 \text{ MJ} = 7.79 \text{ GJ per gram U-235}$$

Energy in 10,000 tons TNT = 338 TJ

$$\frac{338 \text{ TJ}}{1} \cdot \frac{1000 \text{ GJ}}{1 \text{ TJ}} \cdot \frac{1 \text{ g U-235}}{7.79 \text{ GJ}} \cdot \frac{1 \text{ kg}}{1000 \text{ g}} = 43.4 \text{ kg U-235}$$

$$c) E = mc^2 = 43.4 \text{ kg} \cdot (3 \times 10^8 \text{ m/s})^2 = 3.91 \times 10^{18} \text{ J} = 3.91 \text{ EJ}$$

$$\text{Nuclear rxn releases } \frac{338 \text{ TJ}}{3.91 \text{ EJ}} = \frac{338}{3.91 \times 10^6} = 0.0086\% \text{ of energy}$$

2.6 a) Baseball weighs 145 g, thrown at 80 mph = 35.8 m/s

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ m}^2 \cdot \text{kg/s}}{0.145 \text{ kg} \cdot 35.8 \text{ m/s}} = 1.28 \times 10^{-34} \text{ m}$$

$$b) \text{ Kinetic energy} = \frac{1}{2} kT = \frac{1}{2} \cdot 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}} \cdot 300 \text{ K} = 2.07 \times 10^{-21} \text{ J}$$

$$\text{Mass of } N_2 = \frac{28 \text{ g}}{6.02 \times 10^{23}} = 4.65 \times 10^{-23} \text{ g} = 4.65 \times 10^{-26} \text{ kg}$$

$$\text{Set } \frac{1}{2} kT = \frac{1}{2} mv^2 \rightarrow 2.07 \times 10^{-21} \text{ J} = \frac{1}{2} \cdot 4.65 \times 10^{-26} \text{ kg} \cdot v^2$$

$$\hookrightarrow v = 298 \text{ m/s}$$

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34} \text{ m}^2 \cdot \text{kg/s}}{4.65 \times 10^{-26} \text{ kg} \cdot 298 \text{ m/s}} = 4.79 \times 10^{-11} \text{ m} = 47.9 \text{ pm}$$

$$c) PV = nRT$$

$$\text{Pressure} = 1 \text{ atm} = 101325 \text{ Pa}$$

$$\text{Temp} = 300 \text{ K}$$

$$101 \text{ kPa} \cdot V = 1 \text{ mol} \cdot 8.31 \frac{\text{L} \cdot \text{kPa}}{\text{mol} \cdot \text{K}} \cdot 300 \text{ K}$$

$$V = 24.7 \text{ L} = 0.0247 \text{ m}^3 \text{ per mole } N_2$$

$$\rightarrow \frac{0.0247 \text{ m}^3}{1 \text{ mol}} \cdot \frac{1 \text{ mol}}{6.02 \times 10^{23} \text{ molecules}} = 4.10 \times 10^{-26} \text{ m}^3 / \text{molecule}$$

$$\text{Sphere volume} = \frac{4}{3} \pi r^3 = 4.10 \times 10^{-26} \text{ m}^3 \rightarrow r = 2.14 \times 10^{-9} \text{ m}$$

$$\text{Spacing} = 2 \cdot r = \boxed{4.28 \text{ nm}}$$

$$d) v = \sqrt{\frac{kT}{m}}$$

$$\lambda = \frac{h}{m} \cdot \sqrt{\frac{m}{kT}} = h \sqrt{\frac{1}{m \cdot kT}} = 4.28 \times 10^{-9} \text{ m}$$

$$\frac{h^2}{m \cdot kT} = 1.83 \times 10^{-17} \text{ m}^2$$

$$T = \frac{h^2}{m \cdot k \cdot 1.83 \times 10^{-17} \text{ m}^2} = \frac{(6.63 \times 10^{-34} \text{ m}^2 \cdot \text{kg} / \text{s})^2}{4.65 \times 10^{-26} \text{ kg} \cdot 1.38 \times 10^{-23} \frac{\text{J}}{\text{K}} \cdot 1.83 \times 10^{-17} \text{ m}^2}$$

$$= 0.0387 \text{ K} = \boxed{38.7 \text{ mK}}$$

$$\boxed{2.7} \text{ a) } KE = -PE$$

$$\frac{1}{2} m v^2 = \frac{GMm}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$b) c = \sqrt{\frac{2GM}{r}}$$

$$c^2 = \frac{2GM}{r}$$

$$r = \frac{2GM}{c^2}$$

$$c) E = mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{h}{mc}$$

$$d) \frac{2GM}{c^2} = \frac{h}{Mc}$$

$$M^2 = \frac{hc}{2G}$$

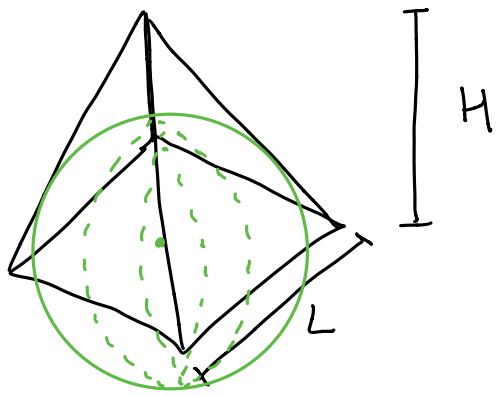
$$M = \sqrt{\frac{hc}{2G}}$$

$$e) r = \frac{2GM}{c^2} = \frac{2G}{c^2} \sqrt{\frac{hc}{2G}} = \sqrt{\frac{4G^2}{c^4} \cdot \frac{hc}{2G}} = \sqrt{\frac{2Gh}{c^3}}$$

$$f) E = Mc^2 = \sqrt{\frac{hc}{2G}} c^2 = \sqrt{\frac{hc^5}{2G}}$$

$$g) f = \frac{E}{h} \rightarrow T = \frac{h}{E} = h \sqrt{\frac{2G}{hc^5}} = \sqrt{\frac{2Gh}{c^5}}$$

2.8



a) Sphere radius = $\frac{L}{2}$