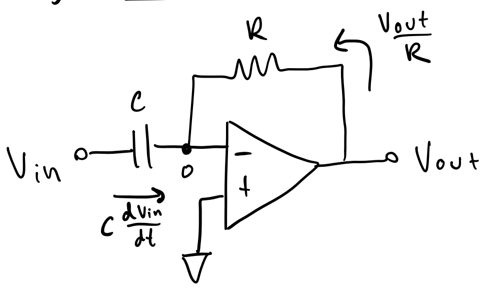
15.1

a) Differentiate:

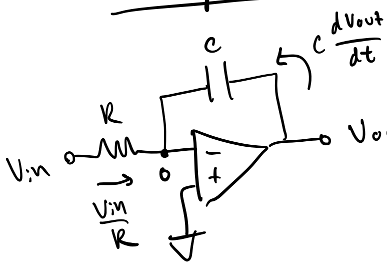


$$C\frac{dV_{in}}{dt} + \frac{V_{out}}{R} = 0$$

$$C\frac{dV_{in}}{dt} = -\frac{V_{out}}{R}$$

$$V_{out} = -RC\frac{dV_{in}}{dt}$$

Integrate:



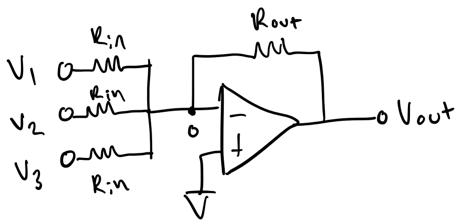
$$\frac{V_{in}}{R} + C\frac{dV_{out}}{dt} = 0$$

$$\frac{V_{in}}{R} = -C\frac{dV_{out}}{dt}$$

$$\frac{dV_{out}}{dt} = -\frac{V_{in}}{RC}$$

$$V_{out} = -\frac{1}{RC}\int V_{in}\,dt$$

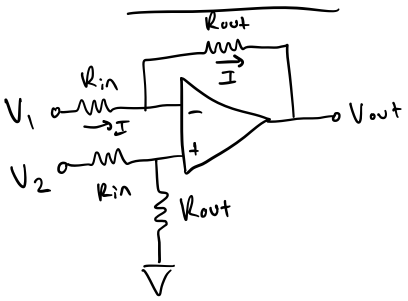
Sum:



$$\frac{V_1}{R_{in}} + \frac{V_2}{R_{in}} + \frac{V_3}{R_{in}} = -\frac{V_{out}}{R_{out}}$$

$$V_{out} = -\frac{R_{out}}{R_{in}}(V_1 + V_2 + V_3)$$

Difference:



$$V_+ = V_2\frac{R_{out}}{R_{in}+R_{out}} = V_-$$

$$V_1 - V_- = IR_{in}$$

$$V_- - V_{out} = IR_{out}$$

$$\frac{V_1 - V_-}{R_{in}} = \frac{V_- - V_{out}}{R_{out}} \rightarrow \frac{V_1}{R_{in}} - \frac{V_-}{R_{in}} = \frac{V_-}{R_{out}} - \frac{V_{out}}{R_{out}}$$

$$\hookrightarrow \frac{V_{out}}{R_{out}} = \frac{V_-}{R_{out}} + \frac{V_-}{R_{in}} - \frac{V_1}{R_{in}}$$

$$\frac{V_{out}}{R_{out}} = V_-\left(\frac{1}{R_{out}} + \frac{1}{R_{in}}\right) - \frac{V_1}{R_{in}}$$

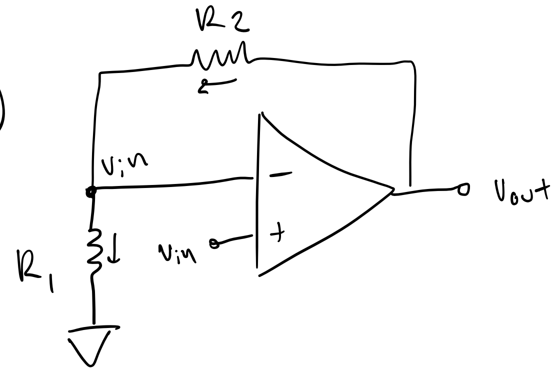
$$\frac{V_{out}}{R_{out}} = V_-\left(\frac{R_{in}+R_{out}}{R_{out}R_{in}}\right) - \frac{V_1}{R_{in}}$$

$$\frac{V_{out}}{R_{out}} = V_2\frac{\cancel{R_{out}}}{\cancel{R_{in}+R_{out}}}\left(\frac{\cancel{R_{in}+R_{out}}}{\cancel{R_{out}}R_{in}}\right) - \frac{V_1}{R_{in}}$$

$$\frac{V_{out}}{R_{out}} = \frac{1}{R_{in}}(V_2 - V_1)$$

$$V_{out} = \frac{R_{out}}{R_{in}}(V_2 - V_1)$$

b)



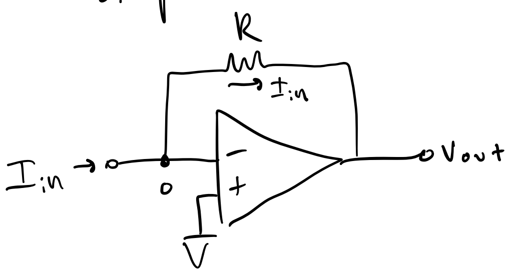
$$\frac{V_{out} - V_{in}}{R_2} = \frac{V_{in}}{R_1}$$

$$V_{out} = \frac{R_2}{R_1} V_{in} + V_{in}$$

$$V_{out} = V_{in}\left(\frac{R_2}{R_1} + 1\right)$$

Disadvantage: gain must be > 1

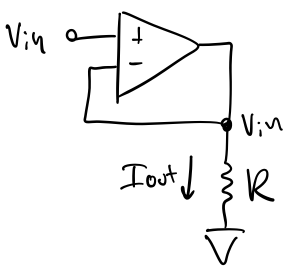
c) Transimpedance:



$$\frac{0 - V_{out}}{R} = I_{in}$$

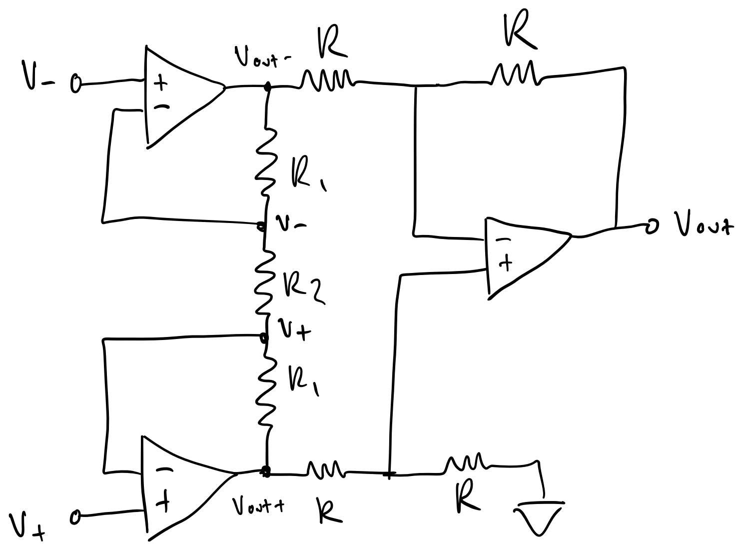
$$V_{out} = -I_{in} R$$

Transconductance:



$$I_{out} = \frac{V_{in}}{R}$$

d)



$$\frac{V_{out-} - V_-}{R_1} + \frac{V_+ - V_-}{R_2} = 0$$

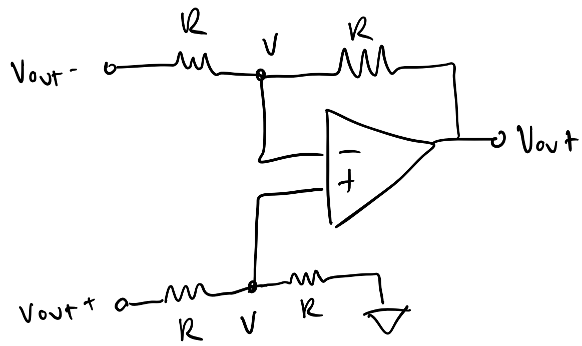
$$\frac{V_{out-} - V_-}{R_1} = \frac{V_- - V_+}{R_2}$$

$$V_{out-} = \frac{R_1}{R_2}(V_- - V_+) + V_-$$

$$\frac{V_{out+} - V_+}{R_1} + \frac{V_- - V_+}{R_2} = 0$$

$$\frac{V_{out+} - V_+}{R_1} = \frac{V_+ - V_-}{R_2}$$

$$V_{out+} = \frac{R_1}{R_2}(V_+ - V_-) + V_+$$



$$\frac{V_{out-} - V}{R} = \frac{V - V_{out}}{R}$$

$$V_{out-} = 2V - V_{out}$$

$$\frac{V_{out+} - V}{R} = \frac{V}{R}$$

$$V_{out+} = 2V$$

$$V_{out-} + V_{out} = V_{out+}$$

$$V_{out} = V_{out+} - V_{out-}$$

$$= \frac{R_1}{R_2}(V_+ - V_-) + V_+ - \frac{R_1}{R_2}(V_- - V_+) - V_-$$

$$= 2\frac{R_1}{R_2}(V_+ - V_-) + V_+ - V_-$$

$$= (V_+ - V_-)\left(2\frac{R_1}{R_2} + 1\right)$$

15.2

GBP = 10 MHz

Open loop DC gain = 100 dB

Inverting amplifier

Plot magnitude and phase of gain vs. frequency as $\frac{R_{out}}{R_{in}}$ varied

$$\omega_1 \approx G_{ol}\,\omega_{ol}$$

$$10\,\text{MHz} = 100\,\text{dB}\;\omega_{ol}$$

$$100 = 20\log_{10}\left(\frac{10\,\text{MHz}}{\omega_{ol}}\right)$$

$$\hookrightarrow 10^5 = \frac{10\,\text{MHz}}{\omega_{ol}} \rightarrow \omega_{ol} = 10\,\text{MHz}$$

$$V_{out} = -\frac{R_{out}}{R_{in}} V_{in} \rightarrow \text{gain} = \frac{R_{out}}{R_{in}}$$

$$G(\omega) = \frac{G_{ol}}{1 + i\frac{\omega}{\omega_{ol}}} = \frac{100\,\text{dB}}{1 + i\frac{\omega}{10\,\text{MHz}}}$$

$$\hookrightarrow G(\omega)\cdot\left(1 + i\frac{\omega}{10\,\text{MHz}}\right) = 100\,\text{dB}$$

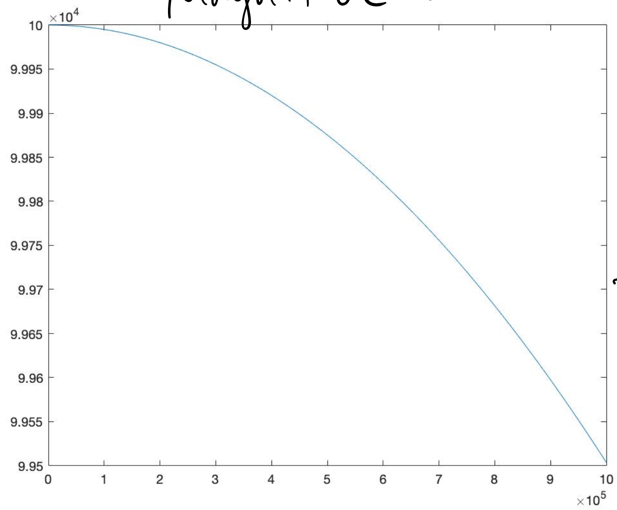
$$100 = 20\log_{10}\left(G(\omega)\cdot\left(1 + i\frac{\omega}{10\,\text{MHz}}\right)\right)$$

$$10^5 = G(\omega)\cdot\left(1 + i\frac{\omega}{10\,\text{MHz}}\right)$$

$$G(\omega) = \frac{10^5}{1 + i\frac{\omega}{10\,\text{MHz}}}$$

Magnitude vs. ω

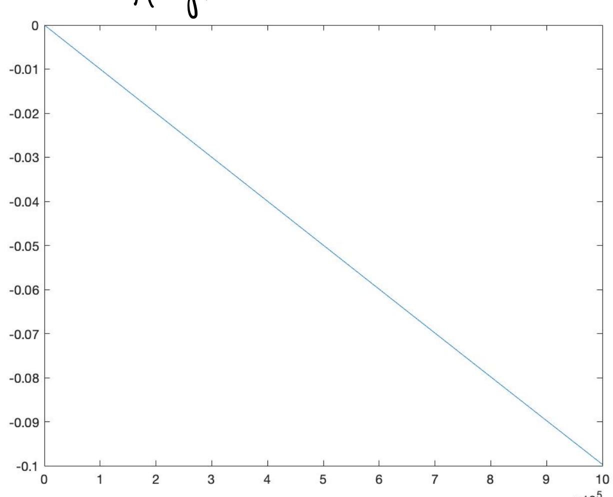
$|G|$



ω

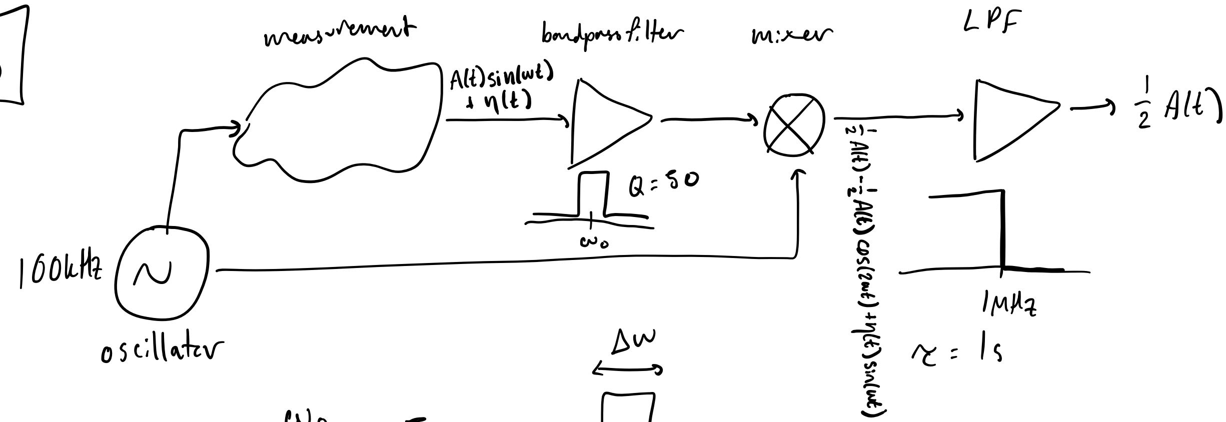
Angle vs. ω

$\angle G$



ω

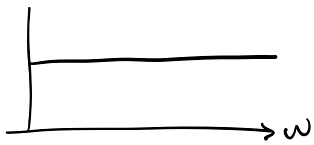
15.3

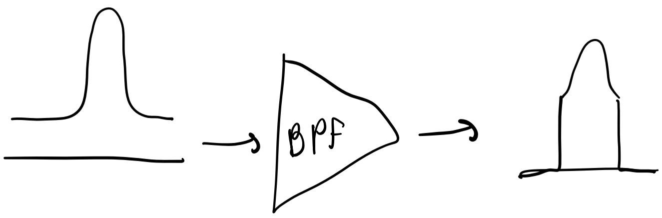


BPF: $Q = \frac{\omega_0}{\Delta\omega} = 50$

$\omega_0 = 100\text{ kHz} \rightarrow \Delta\omega = 2\text{ kHz}$

Noise:





15.5 SNR due to quantization noise : 8, 16 - bit A/D

Averaging of 8-bit to match 16-bit

$$SQNR = 20 \log_{10}(2^Q) \approx 6.02 \cdot Q \text{ dB}$$

where Q = # quantization bits

8-bit → 48.2

16-bit → 96.3