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SELF-REPRODUCTION IN SMALL CELLULAR AUTOMATA

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Self-reproduction in cellular automata is discussed with reference to Langton's criteria as to what constitutes genuine self-reproduction. It is found that it is possible to construct self-reproducing structures that are substantially less complex than that presented by Langton.

1. Introduction

John von Neumann was the first person to design a cellular automaton to model a self-reproducing machine [1]. His structure involved thousands of cells in a two-dimensional array using 29 states per cell. This was significantly simplified by Codd [2], who was able to construct a self-reproducing model using only 8 states per cell. However, even this latter model involves tens of thousands of cells and has never been implemented in an actual simulation.

The above models were universal constructors in the sense that they were designed to construct any machine described on its input tape. Self-reproduction is then a special case where the machine described is the universal constructor itself. In order to construct a simpler machine capable of self-reproduction Langton [3, 4] adopted more liberal criteria as to what constitutes genuine self-reproduction. He dropped the condition that the self-reproducing unit must be capable of universal construction. To rule out trivial cases of "reproduction" Langton specified that the *self* in self-reproduction should be taken seriously and that the construction of a copy should be actively directed by the configuration itself, rather than being merely a consequence of the transition rules. The criterion employed by Langton requires that the information embedded in the cellular automaton should be both translated (i.e. used as instructions to be executed) and transcribed (i.e. copied as uninterpreted data).

Langton then proceeded to construct a self-reproducing automaton that satisfied this criterion. His model is significantly less complex than that of Codd and can readily be simulated on a computer.

Recently Newman [5] estimated a lower limit to the probability of life arising by chance by assuming Langton's model to be near the minimum complexity required for self-reproduction. It is therefore of interest to examine whether Langton's device is indeed a minimal configuration for self-reproduction. In this paper we demonstrate that, with the application of Langton's criterion for self-reproduction, it is possible to construct models which are even smaller than Langton's machine.

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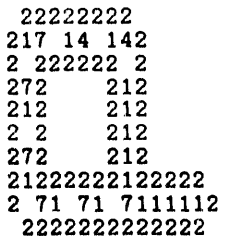

Fig. 1.

2. Langton's automaton

In his design of a self-replicating mechanism Langton considered a two-dimensional array of cells, each cell being in one of eight possible states. The state of each cell at any time is determined by the states of itself and its four nearest neighbors at the previous time-step.

Langton's device is shown in fig. 1. It consists essentially of a signal, guided by two walls, that contains the information necessary to make a copy of itself. The zero state (represented by a blank) is the quiescent state. States 1 and 2 guide the signal: 1 is an element of the data path, 2 is an element of the wall protecting the signal. The remaining five states are used as signals. To specify the direction of the signal the digit following a signal is set to state 0. When a signal approaches a junction it splits into two copies of itself, one alone each path. The data path is lengthened by one unit when a 7–0 signal reaches the end; a left-hand corner is made when two 4–0 signals hit the end in succession. State 3 is used as an intermediate step in turning a left corner, while states 5 and 6 break off the daughter from the parent and initiate the construction of new loops.

With these rules the configuration shown in fig. 1 first extends its arm by six units. Next it turns left, adds another six units, turns left again, adds six more units and turns left a third time. Then it closes in on itself. States 5 and 6 are finally applied to disconnect the new loop and to start the process over again. After 151 time steps we have two loops, each of which starts to form a new loop. The loops continue to reproduce themselves until all the available space is used up.

3. Some simple automata

The question arises whether the above automaton is indeed at or near the minimum possible complexity for self-reproduction, as defined by Langton's criterion.

We note that it is possible to simplify Langton's mechanism by making a few straightforward adjustments. The first simplification is to use just one 4–0 signal to make a left turn. It is feasible then also to drop the intermediate state 3. A second modification is to eliminate the inner wall. Consequently the direction of the signal need not be determined by an x–0 combination, as required by Langton, but by its orientation with respect to the remaining outer wall. A third change is to combine the functions of states 5 and 6 into just one state.

With these alterations we require only six states whose functions are as follows: state 0 still refers to the quiescent state, state 1 defines the data path, state 2 represents the outer wall, 3 is used to extend the path by one unit, state 4 turns a left corner, and state 5 disconnects the newly formed loop and initiates the generation of a further copy.

With a prudent choice of transition rules it was found that it was possible to construct a variety of self-reproductive configurations. One example is the 20 cell structure shown in fig. 2. It produced an exact copy of itself after 46 time units and at 50 time units it is back to its initial arrangement, but with a 90 degree rotation. Thus it fulfills a further requirement of Langton: that the result of true reproduction should be two copies which are identical both physically and behaviorally to the original. Also the general features of the growth were similar to those for Langton's device: first a

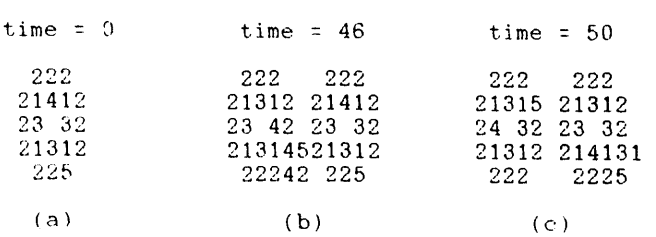

Fig. 2.

Table I
Transition function table for fig. 2

CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I	
00003	1	10000	0	20000	0	30001	0	40003	5	50001	0
00012	2	10001	0	20015	5	30003	0	40022	5	50022	5
00013	1	10004	0	20022	0	30011	0	40035	2	50032	5
00015	4	10033	0	20035	5	30235	3	40043	4	50122	5
00025	4	10043	1	20202	0	302··	5	40212	4	50222	0
00031	5	10325	5	20215	5	312..	5	40232	4	50244	5
00032	3	10421	4	20235	5	3----	1	4024.	4	50322	5
00042	2	10423	4	20252	5			40252	0	50412	4
00121	1	10424	4	2----	2			40325	5	50422	0
00204	2	11142	4					41452	5	5----	2
00324	3	11423	4					4----	1		
00422	2	12234	4								
00532	3	12334	4								
0----	0	12443	4								
		1----	3								

Neighborhoods are read as follows (rotations are not listed):

L T / C / B $R \rightarrow I$

```
time = 0      time = 1      time = 2      time = 3      time = 4      time = 5

 22            22            22            22            22            22
2312          2142          2432          2332          2312          2142
2342          2332          2133          24131         23413         233311
 25            225           225           225           2252          2242

time = 6      time = 7      time = 8      time = 9      time = 10     time = 11

 22            22            22            22            22  1         22
2432          2332          2312 1        2142 3        2432          2332  1
21333         241333        2341331       2334133       21334131      24133413
 22422         22422         22422         224222        224222        2242222


time = 11     time = 13     time = 14     time = 15     time = 16     time = 17

 22            22            22            22  1         22            22    1
2312   1      2142 1        2432 31       2332   1      2312   32     2142 1 12
234133411     23341334      213341332     241334132     234133412     233413342
 2242222       22422222      22422222      2242222       2242222       2242222


time = 18     time = 19     time = 20     time = 21     time = 22     time = 23

 22     2      22  1 22      22    2       22   22       22   22       22   22
2432 3 42     2332   32     2312  132     2142 5112     2432 2142     2332 2432
213341332     241334132     234133412     233413342     213345332     241352132
 2242222       2242222       2242222       2242222       2242222       2242522


time = 24     time = 25     time = 26     time = 27

 22   22       22   22       225  22       235  22
2312 2332     2145 2312     2432 2142     2332 2432
2345 2412     2332 2342     2132 2332     2412 2133
 2242 52       22 2 25       22   225      22   225
```

Fig. 3.

Table II
Transition function table for fig. 3

CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I		CTRBL → I	
00003	1	10003	3	20215	5	31215	1	40242	4
00012	2	10004	0	20235	3	31223	1	40252	0
00013	1	10033	0	20252	5	31233	1	40325	5
00015	2	10043	1	2----	2	31235	5	4----	3
00025	5	10321	3	30001	0	31432	1	50022	5
00031	5	11253	1	30003	0	31452	5	50032	5
00032	3	12453	3	30011	0	3----	3	50212	4
00042	2	1----	4	30012	1			50222	0
0----	0	20000	0	30121	1	40003	5	50322	0
		20015	5	30123	1	40043	4	5----	2
10000	0	20022	0	31122	1	40212	4		
10001	0	20202	0	31123	1	40232	4		

tail is formed; next this tail, or arm, is extended and bent to form a loop; then the loop is closed; finally the offspring is disconnected from its parent and a new cycle is initiated.

The transition rules that were applied to generate the above example are listed in table I. (The C---- rule is the default: it applies to any other combination starting with that value of C which is not listed.)

Since our prime purpose is to examine self-reproduction in configurations of minimal complexity we present a more detailed study of an initial configuration with only 12 cells. As shown in fig. 3, it reproduces itself after 25 time-steps.

After 25 steps the original configuration and the daughter have the exact same form as the initial array. Then, as in the case of Langton's automaton, the daughter forms a new copy towards the right, while the original has turned 90 degrees and makes a copy towards the top of fig. 2. The process continues until all the available space is covered with copies. The new set of transition rules used to generate fig. 3 is listed in table II.

4. Comparison of complexity

The complexity of the automaton is determined by the number of possible states per cell, the number of cells in the configuration, and the number of transition rules.

In Langton's model there were 8 states per cell. If we determine the number of cells by counting only the non-quiescent cells then the number of cells in Langton's initial configuration is 86. Actually, this configuration is generated from a simpler one with only 72 non-quiescent cells. The total number of transition rules given by Langton is 219 but this can be reduced to 108 if we simplify them by applying the default rules 0--- → 1, 1--- → 1, 2--- → 2, 3--- → 1, 4--- → 0, 5--- → 2, 6--- → 1, 7--- → 0. Thus Langton's automaton consists essentially of 8 states per cell, 72 cells, and 108 transition rules (this number includes the 8 default rules), as compared to 6 states, 12 cells, and 57 rules for our simple 12 cell model. This represents a significant reduction in complexity.

Even the 12 cell model is by no means the absolute minimum for self-replication. With a little ingenuity it is no doubt possible to trim the mechanism somewhat further. Note, for example, that with the above rules (i.e., those of table II) the 11 cell configuration shown in fig. 4 fills out to 12 cells and then follows the same pattern as in fig. 3.

```
 22
231
2342
 25
```

Fig. 4.

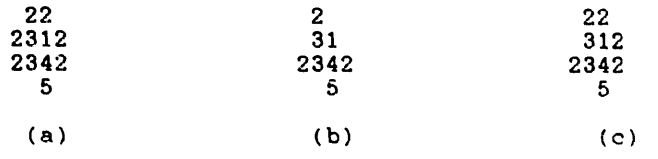

Fig. 5.

With the addition of 2 more rules (for a total of 59) the structure depicted in fig. 5(a) will reproduce itself. These new rules also cause the 8 cell configuration of fig. 5(b) to produce one 11 cell offspring equivalent to that of fig. 5(a). The offspring all make 3 exact copies of themselves (if enough space is available).

If four further transition rules are applied (for a total of 63) then the 10 cell structure of fig. 5(c) will produce 2 exact copies of itself before becoming sterile.

5. Conclusion

We have demonstrated the existence of very simple structures that satisfy all the criteria for self-reproduction as specified by Langton. It was found that only six possible states per cell were required. The minimal configurations found were 10 cells for a structure producing exact copies of itself, and 8 cells for a configuration that generates an 11 cell daughter which in turn produces identical 11 cell offspring. These structures are significantly less complex than the model constructed by Langton.

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