The Physics of Information Technology

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2 Interactions, Units, and Magnitudes

Modern information technology operates over a spectacular range of scales; bits from a memory cell with a size of 10^{-8} meters might be sent 10^{7} meters to a geosynchronous satellite. It is important to be comfortable with the orders of magnitudes and associated interaction mechanisms that are useful in practice. Our first task will be to review the definitions of important units, then survey the types of forces, and finally look at typical numbers in various regimes.

2.1 UNITS

Many powers of ten have been named because it is much easier to say something like "a femtosecond optical pulse" than "a 0.000 000 000 000 001 second optical pulse" when referring to typical phenomena at that scale (a cycle of light takes on the order of a femtosecond). The dizzying growth of our ability to work with large and small systems pushes the bounds of this nomenclature; data from exabyte storage systems is read out into femtofarad memory cells. It is well worth memorizing the prefixes in Table 2.1.

Table 2.1. *Orders of magnitude.*

Magnitude	Prefix	Symbol	Magnitude	Prefix	Symbol
10^{-24}	yocto	y	10^{24}	yotto	Y
10^{-21}	zepto	z	10^{21}	zetta	Z
10^{-18}	atto	a	10^{18}	exa	E
10^{-15}	femto	f	10^{15}	peta	P
10^{-12}	pico	p	10^{12}	tera	T
10^{-9}	nano	n	10^{9}	giga	G
10^{-6}	micro	μ	10^{6}	mega	M
10^{-3}	milli	m	10^{3}	kilo	k
10^{-2}	centi	c	10^{2}	hecto	h
10^{-1}	deci	d	10^{1}	deka	da

Physical quantities must of course be measured in a system of units; there are many alternatives that are matched to different regimes and applications. Because of their interrelationships it is necessary only to define a small number of fundamental quantities to be able to derive all of the other ones. The choice of which fundamental definitions to use changes over time to reflect technological progress; once atomic clocks made it possible to measure time with great *precision* (small variance) and *accuracy* (small bias), it became more reliable to define the meter in terms of time and the speed of light rather than a reference bar kept at the Bureau International des Poids et Mesures (BIPM, `http://www.bipm.org`) in Sevres, France. More recently, a great source of frustration in the metrology community [Girard, 1994] was removed when the definition of the kilogram based on a platinum–iridium cylinder held at BIPM was replaced by one based on fundamental physical constants [Mills *et al*., 2005].

The most common set of base defined quantities in use is the *Système International d'Unités* (*SI*) [BIPM, 2019]. It is based on seven physical constants that were originally measured and are now taken as fixed (there is no uncertainty in these values):

$\Delta\nu_{Cs}$ = 9 192 631 770 Hz: the unperturbed ground state hyperfine transition frequency of the cesium 133 atom
c = 299 792 458 m/s: the speed of light in vacuum
h = 6.626 070 15 $\times 10^{-34}$ J · s: the Planck constant
e = 1.602 176 634 $\times 10^{-19}$ C: the elementary charge
k = 1.380 649 $\times 10^{-23}$ J/K: the Boltzmann constant
N_A = 6.022 140 76 $\times 10^{23}$ mol^{-1}: the Avogadro constant
K_{cd} = 683 lm/W: the luminous efficacy of monochromatic radiation of frequency 540 $\times 10^{12}$ Hz

These constants then define seven base units (which were previously themselves taken as fundamental):

time: *second* (s)
It is defined by taking the fixed numerical value of the cesium frequency $\Delta\nu_{Cs}$, the unperturbed ground-state hyperfine transition frequency of the cesium 133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}.

length: *meter* (m)
It is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299 792 458 when expressed in the unit m · s^{-1}, where the second is defined in terms of $\Delta\nu_{Cs}$.

mass: *kilogram* (kg)
It is defined by taking the fixed numerical value of the Planck constant h to be 6.626 070 15 $\times 10^{-34}$ when expressed in the unit J · s, which is equal to kg · m^2 · s^1, where the metre and the second are defined in terms of c and $\Delta\nu_{Cs}$.

electric current: *ampere* (A)
It is defined by taking the fixed numerical value of the elementary charge e to be 1.602 176 634 $\times 10^{-19}$ when expressed in the unit C, which is equal to A · s, where the second is defined in terms of $\Delta\nu_{Cs}$.

temperature: *kelvin* (K)
It is defined by taking the fixed numerical value of the Boltzmann constant k to be 1.380 649 $\times 10^{-23}$ when expressed in the unit J · K^{-1}, which is equal to kg · m^2 · s^2 · K^{-1}, where the kilogram, meter and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

amount of substance: *mole* (mol)

One mole contains exactly 6.022 140 76 $\times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in the unit mol^{-1} and is called the Avogadro number. The amount of substance, symbol n, of a system is a measure of the number of specified elementary entities. An elementary entity may be an atom, a molecule, an ion, an electron, any other particle or specified group of particles.

luminous intensity: *candela* (cd)

It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency 540 $\times 10^{12}$ Hz, K_{cd}, to be 683 when expressed in the unit $lm \cdot W^{-1}$, which is equal to $cd \cdot sr \cdot W^{-1}$, or $cd \cdot sr \cdot kg^{-1} \cdot m^{-2} \cdot s^3$, where the kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

And then a number of other quantities are defined with these units (most of which we'll be using in coming chapters):

plane angle: *radian* (rad, m/m, the ratio of circular arc length to radius)
solid angle: *steradian* (sr, m^2/m^2, the ratio of spherical area to square radius)
frequency: *hertz* (Hz, s^{-1})
force: *newton* (N, $kg \cdot m \cdot s^{-2}$)
pressure, stress: *pascal* (Pa, $N/m^2 = kg \cdot m^{-1} \cdot s^{-2}$)
energy, work, amount of heat: *joule* (J, $N \cdot m = kg \cdot m^2 \cdot s^{-2}$)
power, radiant flux: *watt* (W, $J/s = kg \cdot m^2 \cdot s^{-3}$)
electric charge: *coulomb* (C, $A \cdot s$)
electric potential difference: *volt* (V, $W/A = kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$)
capacitance: *farad* (F, $C/V = kg^{-1} \cdot m^{-2} \cdot s^4 \cdot A^2$)
electric resistance: *ohm* (, $V/A = kg \cdot m^2 \cdot s^{-3} \cdot A^{-2}$)
electric conductance: *siemens* (S, $A/V = kg^{-1} \cdot m^{-2} \cdot s^3 \cdot A^2$)
magnetic flux: *weber* (Wb, $V \cdot s = kg \cdot m^2 \cdot s^{-2} \cdot A^{-1}$)
magnetic flux density: *tesla* (T, Wb/m 2 = $kg \cdot s^{-2} \cdot A^{-1}$)
inductance: *henry* (H, $Wb/A = kg \cdot m^2 \cdot s^{-2} \cdot A^{-2}$)
Celsius temperature: *degree Celsius* (°C, K = °C+273.15)
luminous flux: *lumen* (lm, cd · sr = cd)
illuminance: *lux* (lx, $lm/m^2 = cd \cdot m^{-2}$)
activity referred to a radionuclide: *becquerel* (Bq, s^{-1})
absorbed dose, kerma: *gray* (Gy, $J/kg = m^2 \cdot s^{-2}$)
dose equivalent: *sievert* (Sv, $J/kg = m^2 \cdot s^{-2}$)
catalytic activity: *katal* (kat, $mol \cdot s^{-1}$)

It is important to pay attention to the units in these definitions. Many errors in calculations can be caught by making sure that the final units are correct, and it can be possible to make a rough estimate of an answer to a problem simply by collecting relevant terms with the right units (this is the subject of *dimensional analysis*). Electromagnetic units are particularly confusing; we will consider them in more detail in Chapter 6. The SI system is also called *MKS* because it bases its units on the meter, the kilogram, and the second. For some problems it will be more convenient to use *CGS* units (based on the

Table 2.2. *Selected conversion factors.*

1 dyne (gm · cm · s^{-2})	=	1×10^{-5} N
1 erg (gm · cm^2 · s^{-2})	=	1×10^{-7} J
1 horsepower (hp)	=	745.7 W
1 atmosphere (atm)	=	101325 Pa
1 ton (short)	=	2000 pounds
	=	907.18474 kg
1 electron volt (eV)	=	$1.602176462\times10^{-19}$ J
1 amu	=	1.66053873×10^{-27} kg
1 angstrom (Å)	=	1×10^{-10} m
1 fermi (fm)	=	1×10^{-15} m
1 parsec (pc)	=	3.085678×10^{16} m
1 mile (mi)	=	1609.344 m
1 foot (ft)	=	0.3048 m
1 inch (in)	=	0.0254 m
1 liter (L)	=	0.001 m^3
1 pound (lb)	=	.45359237 kg
1 pound-force (lbf)	=	4.44822 N

centimeter, the gram, and the second); MKS is more common in engineering and CGS in physics. A number of other units have been defined by characteristic features or by historical practice; some that will be useful later are given in Table 2.2.

It's often more relevant to know the value on one quantity relative to another one, rather than the value itself. The ratio of two values X_1 and X_2, measured in *decibels* (*dB*), is defined to be

$$\mathrm{dB} = 20\log_{10}\frac{X_1}{X_2} \quad . \tag{2.1}$$

If the *power* (energy per time) in two signals is P_1 and P_2, then

$$\mathrm{dB} = 10\log_{10}\frac{P_1}{P_2} \quad . \tag{2.2}$$

This is because the power is the mean square amplitude (Chapter 3), and so to be consistent with equation (2.1) a factor of 2 is brought in to account for the exponent. An increase of 10 db therefore represents a increase by a factor of 10 in the relative power of two signals, or a factor of 3.2 in their values. A change of 3 dB in power is a change by a factor of 2.

The name decibel comes from Bell Labs. Engineers there working on the telephone system found it convenient to measure the gain or loss of devices on a logarithmic scale. Because the log of a product of two numbers is equal to the sum of their logs, this let them find the overall gain of a system by adding the logs of the components, and using logarithms also made it more convenient to express large numbers. They called the base-10 logarithm the *bel* in honor of Alexander Graham Bell; multiplying by 10 to bring up one more significant digit gave them a tenth-bel, or a decibel.

Some decibel reference levels occur so commonly that they are given names; popular ones include:

- dBV measures an electrical signal relative to 1 volt

- *dBm* measures relative to a 1 mW signal. The power will depend on the (usually unspecified) load, which traditionally is 50 Ω for radiofrequency signals and 600 Ω for audio ones (loads will be covered in Chapter 7). In audio recording, this is also called the *Volume Unit* or *VU*.
- *dBspl*, for *Sound Pressure Level* (or just *SPL*), measures sound pressure relative to a reference of 2×10^{-5} Pa, the softest sound that the ear can perceive. The sound of a jet taking off is about 140 dBspl.

Finally, we'll be using a number of fundamental observed constants in nature, including the *gravitational constant* ($G = 6.673(10)\times10^{-11}$ $\text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}$, the *electron mass* ($m_e$) = $9.10938188(72)\times10^{-31}$ kg, and the *proton mass* (m_p) = $1.67262158(13)\times10^{-27}$ kg. In this list the digits in parentheses are the standard deviation uncertainty (see Chapter 3) in the corresponding digits, so that for example the error in the value for G is 0.010×10^{-11} (which, compared to the other constants, is an embarrasingly large uncertainty [Gundlach *et al*., 1996]). Calculating these from first principles remains an open Nobel-prize-worthy problem.

Each fundamental constant can appear in many different types of measurements, and these are done by many different groups, leading to multiple values that unfortunately don't always agree to within their careful error estimates. For this reason, the International Council of Scientific Unions in 1966 formed the *Committee on Data for Science and Technology* (*CODATA*) to do global optimizations over all these data to come up with an internally-consistent set of values [Mohr *et al*., 2016].

2.2 PARTICLES AND FORCES

The world is built out of elementary particles and their interactions. There are a number of natural divisions in organization, energy, and length that occur between the structure of the nucleus of an atom and the structure of the universe; it will be useful to briefly survey this range in order to understand the relevant regimes for present and prospective information technologies.

This story starts with quantum mechanics, the laws that govern things that are very small. Around 1900 Max Planck was led by his inability to explain the spectrum of light from a hot oven to propose that the energy of light is quantized in units of $E = h\nu = hc/\lambda$, where ν is the frequency and λ is the wavelength; $h = 6.626\ldots\times10^{-34}$ J · s is now called *Planck's constant*. From there, in 1905 Einstein introduced the notion of massless photons as the discrete constituents of light, and in 1924 de Broglie suggested that the wavelength relationship applies to massive as well as massless particles by $\lambda = h/p$; λ is the *de Broglie wavelength*, and is a consequence of the *wave–particle duality*: all quantum particles behave as both waves and particles. An electron, or a photon, can diffract like a wave from a periodic grating, but a detector will register the arrival of individual particles. Quantum effects usually become significant when the de Broglie wavelength becomes comparable to the size of an object.

Quantum mechanical particles can be either *fermions* (such as an electron) or *bosons* (such as a photon). Fermions and bosons are as unlike as anything can be in our universe. We will later see that bosons are particles that exist in states that are symmetric under the interchange of particles, they have an integer spin quantum number, and multiple bosons can be in the same quantum state. Fermions have half-integer spin, exist in states that are antisymmetric under particle interchange, and only one fermion can be in a particular quantum state. Spin is an abstract property of a quantum particle, but it behaves just like an angular momentum (as if the particle is spinning).

Particles can interact through four possible forces: *gravitational*, *electromagnetic*, *weak*, and *strong*. The first two are familiar because they have infinite range; the latter two operate on short ranges and are associated with nuclear and subnuclear processes (the characteristic lengths are approximately 10^{-15} m for the strong force and 10^{-18} m for the weak force). The electromagnetic force is so significant because of its strength: if a quantum atom was held together by gravitational forces alone (like a miniature solar system) its size would be on the order of 10^{23} m instead of 10^{-10} m. The macroscopic forces that we feel, such as the hardness of a wall, are transmitted to us by the electromagnetic force through the electrons in our atoms interacting with electrons in the adjoining atoms in the surface, but can be much more simply described in terms of fictitious effective forces ("the wall is hard").

All forces were originally thought to be transmitted by an intervening medium, the long-sought *ether* for electromagnetic forces. We now understand that forces operate by the exchange of spin-1 gauge bosons – the photon for the electromagnetic interaction (electric and magnetic fields), the $W^{\pm}$ and Z^0 bosons for the weak interaction, and eight gluons for the strong interaction (there is not yet a successful quantum theory of gravity). *Quantum ElectroDynamics* (*QED*) is the theory of the quantum electromagnetic interaction, and *Quantum ChromoDynamics* (*QCD*) the theory of the strong interaction. The weak and electromagnetic interactions are united in the *electroweak theory*, which, along with QCD is the basis for the *Standard Model*, the current summary of our understanding of particle physics. This amalgam of experimental observations and theoretical inferences successfully predicts most observed behavior extremely accurately, with two important catches: the theory has 20 or so adjustable parameters that must be determined from experiments, and it cannot explain gravitation. *String theory* [Giveon & Kutasov, 1999], a reformulation of particle theory that starts from loops rather than points as the primitive mathematical entity, appears to address both these limitations, and so is of intense interest in the theoretical physics community even in the absence of experimentally testable predictions.

The most fundamental massive particles that we are aware of are the *quarks* and *leptons*. There's no reason to assume that there's nothing below them (i.e., turtles all the way down); there's just not a compelling reason right now to believe that there is. Quarks and leptons appear in the scattering experiments used to study particle physics to be point-particles without internal structure, and are spin-1/2 fermions. The leptons interact through the electromagnetic and weak interactions, and come in pairs: the *electron* and the electron *neutrino* (e^-, ν_e), the *muon* and its neutrino (μ^-, ν_μ), and the *tau lepton* and its neutrino (τ^-, ν_τ). Muons and tau leptons are unstable, and therefore are seen only in accelerators, particle decay products, and cosmic rays. Because neutrinos interact only through the weak force, they can pass unhindered though a light-year of lead. But they are profoundly important for the energy balance of the universe, and if they have mass [Fukuda, 1998] it will have enormous implications for the fate of the universe. Quarks interact through the strong as well as weak and electromagnetic interactions, and they come in pairs: *up* and *down*, *charm* and *strange*, and *top* and *bottom*. These fanciful names are just labels for the underlying abstract states. The first member of each pair has charge $+2/3$, the second member has charge $-1/3$, and each charge flavor comes in three colors (once again, flavor and color are just descriptive names for quantum numbers).

Quarks combine to form *hadrons*; the best-known of which are the two *nucleons*. A proton comprises two ups and a down, and the neutron an up and two downs. The nucleons, along with their excited states, are called *baryons* and are fermions. Transitions between baryon states can absorb or emit spin-1 boson hadrons, called *mesons*. The size of hadrons is on the order of 10^{-15} m, and the energy difference between excited states is on the order of 10^9 electron volts (1 GeV).

The nucleus of an atom is made up of some number of protons and neutrons, bound into ground and excited states by the strong interaction. Typical nuclear sizes are on the order of 10^{-14} m, and energies for nuclear excitations are on the order of 10^6 eV (1 MeV). Atoms consist of a nucleus and electrons bound by the electromagnetic interaction; typical sizes are on the order of 1 ångstrom (Å, 10^{-10} m) and the energy difference between states is on the order of 1 eV. Notice the large difference in size between the atom and the nucleus: atoms are mostly empty space. Atoms can exist in different *isotopes* that have the same number of protons but differing numbers of neutrons, and *ions* are atoms that have had electrons removed or added.

Atoms can bond to form molecules; bond energies are on the order of 1 eV and bond lengths are on the order of 1 Å. Molecular sizes range from simple diatomic molecules up to enormous biological molecules with 10^6–10^9 atoms. Large molecules fold into complex shapes; this progresses from the *primary structure* of a coding sequence, to the *secondary structure* of geometrical motifs, to the *tertiary structure* of functional units, to the *quaternary structure* of molecular assemblies [Goodsell, 2009]. Predicting this folding is one of the most difficult challenges in molecular biology [R.Evans *et al*., 2018].

Macroscopic materials are described by the arrangement of their constituent atoms, and include crystals (which have complete long-range ordering), liquids and glasses (which have short-range order but little long-range order), and gases (which have little short-range order). There are also very interesting intermediate cases, such as quasiperiodic alloys called *quasicrystals* that have deterministic translational order without translational periodicity [DiVincenzo & Steinhardt, 1991], and *liquid crystals* that maintain orientational but not translational ordering [Chandrasekhar, 1992]. Most solids do not contain just a single phase; there are usually defects and boundaries between different kinds of domains.

The atomic weight of an element is equal to the number of grams equal to one mole ($N_A \approx 10^{23}$) of atoms. It is approximately equal to the number of protons and neutrons in an atom, but differs because of the mix of naturally occuring isotopes. 22.4 liters of an ideal gas at a pressure of 1 atmosphere and at room temperature will also contain a mole of atoms. The structure of a material at more fundamental levels will be invisible and can be ignored unless energies are larger than its characteristic excitations. Although we will rarely need to descend below atomic structure, there are a number of important applications of nuclear transitions, such as nuclear power and the use of nuclear probes to characterize materials.

2.3 ORDERS OF MAGNITUDE

Understanding what is possible and what is preposterous requires being familiar with the range of meaningful numbers for each unit; the following lists include some significant ones:

Time

10^{-43} s: the Planck time (Problem 2.7)
10^{-15} s: this is the period of visible light, and a typical time scale for chemical reactions
10^{-12} s: shortest logic gate delay
10^{-9} s: atomic excitations and molecular rotations typically have lifetimes on the order of nanoseconds, and this is a characteristic computer clock cycle
10^{-3} s: the shortest time difference that is consciously perceptible by people
10^{17} s: the approximate age of the observable universe

Power and Energy

1 eV: atomic excitations
10^{6} eV: nuclear excitations
10^{9} eV: subnuclear excitations
10^{28} eV: the Planck energy
10 W: laptop computer
100 W: workstation; human
1000 W: house
10^{4} W: car
10^{5} W: building
10^{7} W: supercomputer
10^{26} W: luminosity of the sun
10^{-12} W/m^2: softest sound that can be heard
1 W/m^2: loudest sound that can be tolerated
10^{7} J/kg: energy density of food
10^{9} J: energy in a ton of TNT
10^{20} J: energy consumption in the US per year

Temperature

10^{-9} K: laser-cooled gas
2.75 K: microwave background radiation from the Big Bang
77 K: liquid nitrogen
300 K: room temperature
6000 K: surface of the sun

Mass

10^{-27} kg: proton mass
10^{-12} kg: typical cell
10^{-5} kg: small insect

10^{16} kg: Earth's biomass
5.98×10^{24} kg: the mass of the Earth
10^{42} kg: approximate mass of the Milky Way

Length

10^{-35} m: the Planck distance
10^{-15} m: size of a proton
10^{-10} m: size of an atom
4×10^{5} m: height of a Low Earth Orbit satellite above the surface
6.378×10^{6} m: radius of the Earth
4×10^{7} m: height of a geosynchronous satellite above the equator
10^{11} m: distance from the Earth to the Sun
10^{20} m: Milky Way radius
10^{26} m: size of the observable universe

Electromagnetic spectrum

< 0.1 Å: gamma rays
0.1–100 Å: X-rays
100–4000 Å: UV (atomic ionization energy)
4000–7000 Å: visible (this coincides with a transmission band through the atmosphere, and corresponds to 10^{14}–10^{15} Hz)
0.7–100 μm: IR (thermal radiation)
0.01–10 cm: microwave (GHz)
0.1–10^{3} m: radio (MHz–kHz)

Magnetic and Electric Fields

10^{-12} tesla: magnetic field needed for radio reception
10^{-6} tesla: magnetic field generated by a cordless phone
3×10^{-5} tesla: magnetic field at the Earth's surface
20 tesla: large superconducting/normal hybrid magnet
10^{4} A: lightning bolt current
10^{8} V: potential across a lightning bolt
3×10^{6} V/m: breakdown voltage in air

Number

10^{5}: number of DNA bases in a bacteriophage
4×10^{6}: bytes in the Bible
10^{9}: number of DNA bases in a mammal
10^{13}: number of synapses in the human cortex
10^{21}: bytes passing through the Internet in 2016
10^{80}: approximate number of atoms in the universe

2.4 SELECTED REFERENCES

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[Morrison & Morrison, 1982] Morrison, Philip, & Morrison, Phylis. (1982). *Powers Of Ten: A Book About the Relative Size of Things*. Redding: Scientific American Library.

The Morrisons provide a marvelous tour through the characteristic phenomena at many length scales.

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A nice introduction to particles and forces.

2.5 PROBLEMS

(2.1) (*a*) How many atoms are there in a yoctomole?

(*b*) How many seconds are there in a nanocentury? Is the value near that of any important constants?

(2.2) A large data storage system holds on the order of an exabyte. How tall would a 1 exabyte stack of DVDs be? How does that compare to the distance from Earth to space?

(2.3) If all the atoms in our universe were used to write an enormous binary number, using one atom per bit, what would that number be (converted to base 10)?

(2.4) Compare the gravitational acceleration due to the mass of the Earth at its surface to that produced by a 1 kg mass at a distance of 1 m. Express their ratio in decibels.

(2.5) (*a*) Approximately estimate the chemical energy in a ton of TNT. You can assume that nitrogen is the primary component; think about what kind of energy is released in a chemical reaction, where it is stored, and how much there is.

(*b*) Estimate how much uranium would be needed to make a nuclear explosion equal to the energy in a chemical explosion in 10 000 tons of TNT (once again, think about where the energy is stored).

(*c*) Compare this to the *rest mass energy* $E = mc^2$ of that amount of material (Chapter 15), which gives the maximum amount of energy that could be liberated from it.

(2.6) (*a*) What is the approximate de Broglie wavelength of a thrown baseball?

(*b*) Of a molecule of nitrogen gas at room temperature and pressure? (This requires either the result of Section 3.4.2, or dimensional analysis.)

(*c*) What is the typical distance between the molecules in this gas?

(*d*) If the volume of the gas is kept constant as it is cooled, at what temperature does the wavelength become comparable to the distance between the molecules?

(2.7) (*a*) The potential energy of a mass m a distance r from a mass M is $-GMm/r$. What is the *escape velocity* required to climb out of that potential?

(*b*) Since nothing can travel faster than the speed of light (Chapter 15), what is the radius within which nothing can escape from the mass?

(*c*) If the rest energy of a mass M is converted into a photon, what is its wavelength?

(*d*) For what mass does its equivalent wavelength equal the size within which light cannot escape?

(*e*) What is the corresponding size?

(*f*) What is the energy?

(*g*) What is the period?

(2.8) Consider a pyramid of height H and a square base of side length L. A sphere is placed so that its center is at the center of the square at the base of the pyramid, and so that it is tangent to all of the edges of the pyramid (intersecting each edge at just one point).

(*a*) How high is the pyramid in terms of L?

(*b*) What is the volume of the space common to the sphere and the pyramid?

(This question comes from an entrance examination for humanities students at Tokyo University [*Economist*, 1993].)