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PSET 1

2.1
a.

yocto = 10^{-24}
mole = $6.022\ 140\ 76\ \times\ 10^{23}$
yoctomole ≈ 0.6 → So zero atoms.

b.

nano = 10^{-9}
century = $100\ yr = 8760 * 10^2 hour = 3.15 * 10^9 s$
nanocentury = 3.15 seconds. This is close to π, but I think that is meaningless.

2.2

Exa = 10^{18}
Disc thickness = 1.2mm
Let 1 DVD be $5 * 10^9$ bytes
So:

$$\frac{2.4 * 10^{-3} m}{10^{10} bytes} = \frac{2.4 * 10^5 m}{10^{18} bytes}$$

It is 80 km to space and this is 240 km. So it's quite tall.

2.3

10^{80} atoms → The biggest number is $2^{10^{80}} - 1$. Obviously, the minus 1 is very important.

2.4

$$g = 9.8\ m/s$$
$$F = G * \frac{m_1 m_2}{r^2}$$
$$a_{1kg_{1m}} = G = 6.67 * 10^{-11} m/s^2$$
$$20 \log \frac{g}{a_{1kg_{1m}}} = \sim 220 dB$$

2.5
a.

Atomic excitation: 1eV

TNT is $C_7H_5N_3O_6$. The molar mass is 227 g/mol.
1 ton = 907185g.

$$\frac{1mol_{TNT}}{227g} \approx \frac{4*10^3 mol_{TNT}}{907185g}$$

4k moles of TNT contains 12k moles N, or $12*10^3*6*10^{23} = 72*10^{26}$ atoms of Nitrogen.
Assuming 1eV of energy is released per Nitrogen:

$$1eV = 1.6*10^{-19}J$$

$$72*1.6*10^{26}*10^{-19} = 115*10^7 \approx 10^9 J$$

Which matches the figure given in the text book.

b.

$$\frac{10^9 J}{1\ ton\ TNT} = \frac{10^{13} J}{10^4\ ton\ TNT}$$

$10^6 eV$ in nuclear excitations, or $1.6*10^{-13}J$

$$\frac{1.6*10^{-13}J}{1\ nuclear\ excitation} = \frac{10^{13}J}{6.25*10^{25} nuclear\ excitations}$$

Uranium-235 is the normal explode-y one, so making what I think is the incorrect assumption that we have 235 nuclear excitations per uranium atom:

$$\frac{6.25*10^{25} nuclear\ excitations}{235} \rightarrow 2.6*10^{23} atoms$$

This is about half a mole, so this comes out to ~0.18 kg of uranium.

c.

$$E = mc^2 = 0.18*9*10^{16} = 162*10^{16}J = 1.6*10^{18}J$$

Five orders of magnitude, crazy stuff.

2.6

a.

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Let v = 10 m/s. A gentle toss.
m = 0.14kg, h = $6.626*10^{-34}$

$$\lambda_{baseball} \approx 4.7*10^{-34}m$$

b.
N_2 molar mass = 28g/mol

$$\frac{28g}{mol}*\frac{mol}{6.022*10^{23}} = 4.6*10^{-23}g = 4.6*10^{-26}kg$$

$$\frac{kT}{2} = \frac{1}{2}mv_i^2$$
$$v_i = \sqrt{\frac{kT}{m}}$$
$$v^2 = v_x^2 + v_y^2 + v_z^2 = \frac{3kT}{m}$$
$$k = 1.38 * 10^{-23}\frac{J}{K}, T = 300K$$
$$\lambda_{N2} = \frac{h}{m * \sqrt{\frac{3kT}{m}}} = 2.9 * 10^{-11}[m]$$

c.

$$PV = nRT$$
$$\frac{V}{n} = \frac{RT}{P} = 8 * \frac{300}{10^5} = 2.4 * 10^{-2}[m^3/mol]$$

Volume per atom: $\approx 4 * 10^{-26}$

Let's say we have 8 atoms arranged in a cube.

Volume $= 32 * 10^{-26} = d^3$

So $d \approx 10^{-9}m$

d.

whoops this is wrong!

2.7

a.

$$V = -GMm/r$$
$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$
$$v = \sqrt{\frac{2GM}{r}}$$

b.

$$3 * 10^8 = \sqrt{2 * 6.67 * 10^{-11} * \frac{M}{r}}$$
$$r = 1.4 * 10^{-27} * M$$

Aka,

$$r = \frac{2GM}{c^2}$$

c.

$$E = Mc^2 = \frac{hc}{\lambda}$$
$$\lambda = \frac{h}{Mc}$$

d.

$$\rightarrow 1.4 * 10^{-27} * M = 6.6 * \frac{10^{-34}}{3 * 10^8}$$
$$M = 4.5 * 10^{-8}$$

e.

$$\lambda = \frac{(6.6 * 10^{-34})}{3 * 10^8 * 4.5 * 10^{-8}} \approx 0.5 * 10^{-34}[m]$$

Wow! That's small!

f.

$$E = \frac{hc}{\lambda} = 36 * 10^8 J$$

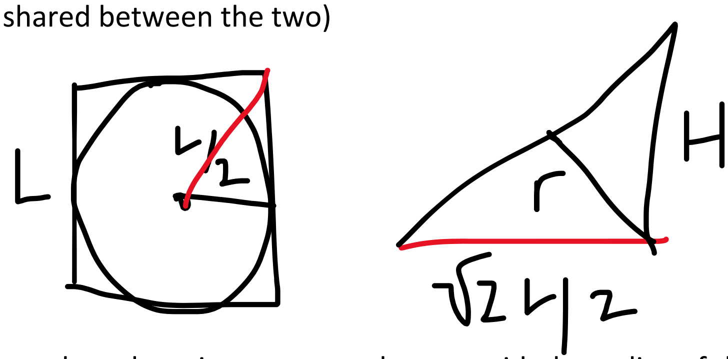
g.

$$f = \frac{3 * 10^8}{0.5 * 10^{-34}} = 6 * 10^{42}$$
$$T = 0.16 * 10^{-42} s$$

2.8

a.

Let's look at the top-down projection as well as the orthogonal-to-diagonal projection (red line is shared between the two)

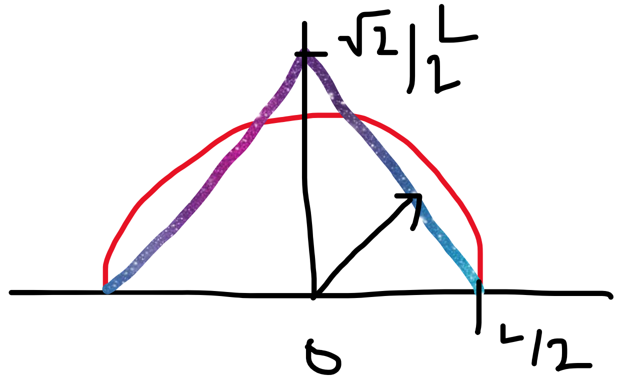


Since the sphere is tangent to the pyramid, the radius of the sphere is perpendicular to the pyramid slant at that place. Given that the height and the base are also orthogonal, we know that r necessarily bisects the angle, and so, via triangles, $H = \frac{\sqrt{2}L}{2}$.

b.

I think there are a couple of ways to do this. We can start with the integral version, but I suspect we can do this without calculus. Both of these are going to start with the same vaguely round about process.

The intersection volume can be described as that of a half-sphere with four end caps slice off by the planes of the pyramid. Frankly, I don't remember how to set this up in 3D space, so we'll look at the project of the sphere and pyramid head on from one of the sides of the pyramid:



The distance from the center of the sphere to the pyramid wall is the same as the shortest path from the line that pass through the pyramid height H and base point to the origin.

$$y = -\sqrt{2}x + \frac{\sqrt{2}}{2}L$$

This is the equation of the line. The distance to the origin is then:

$$dist_0 = \frac{\left|0 + 0 - \frac{\sqrt{2}}{2}L\right|}{\sqrt{2+1}} = \frac{\sqrt{2}}{2\sqrt{3}}L = \frac{L}{\sqrt{6}}$$

Then, (changing our frame of reference), the volume of the cap is the integral of the sphere's cross-sections from $\frac{L}{\sqrt{6}}$ to $\frac{L}{2}$.

So, switch to R for temporary convenience:

$$V_{cap} = \int_{\frac{L}{\sqrt{6}}}^{\frac{L}{2}} \pi(R^2 - x^2)dx = \pi R^2 x - \frac{\pi x^3}{3}\Bigg|_{\frac{L}{\sqrt{6}}}^{\frac{L}{2}} = \frac{\pi L^3}{12}(1 - \frac{7}{36\sqrt{6}})$$

The sphere's volume is $V_{sphere} = \frac{\frac{4}{3}\pi L^3}{8}$

$$V_{intersect} = \frac{\pi L^3}{6} - \frac{\pi L^3}{3}\left(1 - \frac{7}{36\sqrt{6}}\right)$$

Without calculus, we can do this easily if we happen to remember the formula for the volume of a sphere cap. This is:

$$V_{cap} = \frac{1}{3}\pi h^2(3R - h)$$

Where h is $R - dist_0$. This will get us back around to the same place. But let's say we don't remember this formula! Can we geometrically reason this out without using a calculus derivation? The answer is if we don't remember this formula we probably also don't remember what we would need to know to get us out this hole without calculus. But! In keeping with the emotional theme of this problem set, let's do some estimations and see where we end up.

For $L = 1$:

$$V_{intersectReal} \approx 0.01$$

This seems quite small, so maybe there's been a mistake.

Let's make the incorrect approximation that the volume of the sphere sector (the cone + the little hat) is given by:

$$V_{sectorFake} = \frac{2}{3}\pi R^3 \theta$$
$$V_{capFake} = V_{sectorFake} - V_{cone} = \frac{2}{3}R^3\left(\theta - \sin^2\frac{\theta}{4}\right)$$
$$h = R - dist_0 \approx 0.1$$
$$\theta \approx 1.287rad$$
$$V_{capFake} \approx 0.09$$
$$V_{sphereHalf} \approx 0.26$$
$$V_{intersectFake} \approx -0.08$$

Well that's a problem, but I'm arguably not actually very far off :p