Miana Smith
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PSET 9

12.1
a.

We must know the luminous efficacy (lm/W) of the projector. Which so far as I can tell, we do not. Wikipedia says the luminous efficacy of a 5-16W LED screw base lamp is between 75-210 lm/W, so let's say it's 200.

Then: $\frac{1000[lm]}{200\left[\frac{lm}{W}\right]} = 5W$

b.

The text says that the eye can resolve a spatial frequency of 60 cycles per degree or 0.00029 radians. For a very large distance, we can say that the arc length swept by the angle is approximately equal to the height swept by this angle; i.e. a small sine approximation. Then,

$$dot_{1m} = 1 * 2.9 * 10^{-4}m$$

Is the minimum resolvable dot size at 1m. Converting to dpi at 1 meter:

$$\frac{dot_{1m}}{2.9 * 10^{-4}m * 39.37} = 0.0087 * 10^4 = 87[dpi]$$

What about closer?

$$dot_{24"} = 0.0069"$$

Or,

$$\approx 145dpi$$

Seems low, tbh.

12.2
a.

$$U = \frac{8\pi h\nu^3}{c^3\left(e^{\frac{h\nu}{kT}} - 1\right)}$$

We are ~310 K.

$$\frac{\partial U}{\partial \nu} = -\frac{\left(8h\nu^2\pi\left(3Tk - 3Tc^3ke^{\frac{h\nu}{Tk}} + c^3h\nu e^{\frac{h\nu}{Tk}}\right)\right)}{Tk\left(c^3e^{\left(\frac{h\nu}{Tk}\right)} - 1\right)^2}$$

Thanks, Matlab!

$$0 = -\frac{\left(8hv^2\pi\left(3Tk - 3Tc^3ke^{\frac{hv}{Tk}} + c^3hve^{\frac{hv}{Tk}}\right)\right)}{Tk\left(c^3e^{\left(\frac{hv}{Tk}\right)} - 1\right)^2}$$

$$0 = 3Tk - 3Tkc^3e^{\frac{hv}{Tk}} + c^3hve^{\frac{hv}{Tk}}$$

$$0 = 3Tk + e^{\frac{hv}{Tk}}(-3Tkc^3 + c^3hv)$$

$$\frac{3Tk}{c^3(3Tk - hv)} = e^{\frac{hv}{Tk}}$$

This is a mess whose solution requires the Lambert W, per Matlab, so I'm evaluating there. We get:

$$v_{max} = 1.9357E + 13$$

So:

$$\lambda_{peak} = 1.5446E - 05[m]$$

And for T=2.74:

$$\lambda_{peak} = 0.0017$$

b.

What's red?

$$\lambda = 650$$

$$v = 4.62 * 10^{14}$$

This comes out to:

$$T \approx 7400[K]$$

c.

Let's say body surface area is 1.6m^2.

Stefan-Boltzmann gives us:

$$R = \sigma T^4$$

$$R = 5.67 * 10^{-8} * (310)\text{^}4 = 523$$

$$P = R * 1.6 = 837\ [W]$$

Let's try another estimator to ballpark this.

$$2000[kcal\ per\ day] = 8368000[J\ per\ day]$$

$$P = 96.8519\left[\frac{J}{s}\right]$$

There's an order of magnitude difference, which is very suspect! I think the emissivity based approach is more wrong!

12.3

a.

For Calcite: $n_{slow} = 1.658$ and $n_{fast} = 1.486$

Dropping the phase term, 90˚ rotation means:

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}' = \begin{pmatrix} -E_y \\ E_x \end{pmatrix}$$

So:

$$\begin{pmatrix} -E_y \\ E_x \end{pmatrix} = \begin{pmatrix} e^{-i\delta} & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix}$$
$$-E_y = e^{-i\delta} E_x$$
$$E_x = e^{i\delta} E_y$$
$$\delta = (n_{slow} - n_{fast}) \frac{\omega d}{2c}$$

We want to flip-flop the wave, but not alter it, so the condition for that is:

$$e^{i\delta} = 1$$

Which implies:

$$\delta = \frac{\pi}{2}$$
$$\frac{2c\delta}{2\pi f (n_{slow} - n_{fast})} = d$$

$$\frac{2(600 * 10^{-9})\,\pi/2}{2\pi(0.172)} = d$$
$$d = 1.7 * 10^{-6}$$

b.

Circular polarization means that we're constantly rotating but the magnitude isn't changing. Or at least that's what I'm getting from Wikipedia :p i.e. the Jones vector is:

The text says:

$$\frac{E_y}{E_x} = i$$

So:

$$\frac{E_y e^{-i\delta}}{E_x e^{i\delta}} = i$$

Purely rotational so Ey=Ex.

$$e^{-2i\delta} = i$$
$$-2\delta = \pi/2$$
$$\delta = \frac{\pi}{4}$$

So:

$$d = 8.5 * 10^{-7}$$

c.

No more energy.

12.4

For KPV:

$$\rho_x - \rho_y = \frac{1}{c}\omega n_0^3 r_{63} E_z l = \frac{1}{c}\omega n_0^3 r_{63} V$$

$$r_{63} = 10.6 * 10^{-12}$$
$$n_0 = 1.51$$
$$\pi = \frac{1}{700 * 10^{-9}} * 1.51^3 * 10.6 * 10^{-12} * V$$
$$V = \frac{\pi}{\frac{1}{700 * 10^{-9}} * 1.51^3 * 10.6 * 10^{-12}}$$
$$V \approx 60[kV]$$

That seems big? Dropped a 2 pi in there from the angular frequency, not sure that'll fix it, but fyi.

12.5

I no longer have the energy for this! Though did I ever?

a.

b.

c.