$\boxed{6.1}$ $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

LHS: $\vec{A} \times \Big(\underbrace{(B_2C_3 - B_3C_2)}_{D_1}\hat{x}_1 + \underbrace{(B_3C_1 - B_1C_3)}_{D_2}\hat{x}_2 + \underbrace{(B_1C_2 - B_2C_1)}_{D_3}\hat{x}_3 \Big)$

$$= \vec{A} \times \vec{D} = (A_2D_3 - A_3D_2)\hat{x}_1 + (A_3D_1 - A_1D_3)\hat{x}_2 + (A_1D_2 - A_2D_1)\hat{x}_3$$

$$\begin{aligned}
= &\ \hat{x}_1 \cdot \big(A_2(B_1C_2 - B_2C_1) - A_3(B_3C_1 - B_1C_3)\big) \\
&+ \hat{x}_2 \cdot \big(A_3(B_2C_3 - B_3C_2) - A_1(B_1C_2 - B_2C_1)\big) \\
&+ \hat{x}_3 \cdot \big(A_1(B_3C_1 - B_1C_3) - A_2(B_2C_3 - B_3C_2)\big)
\end{aligned}$$

$$\begin{aligned}
= &\ \hat{x}_1 \cdot (A_2B_1C_2 - A_2B_2C_1 - A_3B_3C_1 + A_3B_1C_3) \\
&+ \hat{x}_2 \cdot (A_3B_2C_3 - A_3B_3C_2 - A_1B_1C_2 + A_1B_2C_1) \\
&+ \hat{x}_3 \cdot (A_1B_3C_1 - A_1B_1C_3 - A_2B_2C_3 + A_2B_3C_2)
\end{aligned}$$

equal

RHS: $\vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

$$\begin{aligned}
= &\ (B_1\hat{x}_1 + B_2\hat{x}_2 + B_3\hat{x}_3)(A_1C_1 + A_2C_2 + A_3C_3) \\
&- (C_1\hat{x}_1 + C_2\hat{x}_2 + C_3\hat{x}_3)(A_1B_1 + A_2B_2 + A_3B_3)
\end{aligned}$$

$$\begin{aligned}
= &\ \hat{x}_1 \big(\cancel{A_1B_1C_1} + A_2B_1C_2 + A_3B_1C_3 \\
&\qquad - \cancel{A_1B_1C_1} - A_2B_2C_1 - A_3B_3C_1\big) \\
&+ \hat{x}_2 \big(A_1B_2C_1 + \cancel{A_2B_2C_2} + A_3B_2C_3 \\
&\qquad - A_1B_1C_2 - \cancel{A_2B_2C_2} - A_3B_3C_2\big) \\
&+ \hat{x}_3 \big(A_1B_3C_1 + A_2B_3C_2 + \cancel{A_3B_3C_3} \\
&\qquad - A_1B_1C_3 - A_2B_2C_3 - \cancel{A_3B_3C_3}\big)
\end{aligned}$$

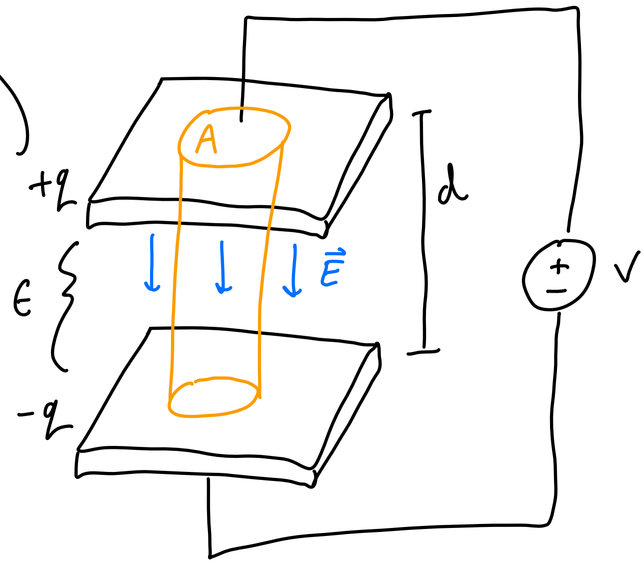
$$\begin{aligned}
= &\ \hat{x}_1 (A_2B_1C_2 + A_3B_1C_3 - A_2B_2C_1 - A_3B_3C_1) \\
&+ \hat{x}_2 (A_1B_2C_1 + A_3B_2C_3 - A_1B_1C_2 - A_3B_3C_2) \\
&+ \hat{x}_3 (A_1B_3C_1 + A_2B_3C_2 - A_1B_1C_3 - A_2B_2C_3)
\end{aligned}$$

$= \text{LHS}$ ✓

Show $\underset{\vec{A}}{\nabla} \times (\underset{\vec{B}}{\nabla} \times \underset{\vec{C}}{\vec{E}}) = \underset{\vec{B}}{\nabla}(\underset{\vec{A}}{\nabla} \cdot \underset{\vec{C}}{\vec{E}}) - \underbrace{\nabla^2 \vec{E}}_{\underset{\vec{C}}{\vec{E}}(\underset{\vec{A}}{\nabla} \cdot \underset{\vec{B}}{\nabla})}$

Matches BAC − CAB rule ✓

$\boxed{6.2}$ a)



$$\int_S \vec{E} \cdot d\vec{A} = \int_V \nabla \cdot \vec{E} \, dV$$

$$EA = \frac{q}{\epsilon} \rightarrow E = \frac{q}{\epsilon A}$$

$$E = \frac{V}{d} = \frac{q}{\epsilon A}$$

$$\hookrightarrow C = \frac{q}{V} = \boxed{\frac{\epsilon A}{d}}$$

b) $i = \frac{dq}{dt}$

$= \frac{d}{dt}(Cv)$

$= C \frac{dv}{dt}$

$q = \int i \, dt$

$v = \frac{1}{C} \int i \, dt$

c) $u = \frac{\text{energy}}{\text{volume}} = \frac{1}{2}\epsilon E^2$

$$\text{energy} = \int u\, dV = \iint u\, dA\, d\ell = \frac{1}{2}\epsilon E^2 Ad$$

$$E = \frac{V}{d} \rightarrow \text{energy} = \frac{1}{2}\epsilon \frac{V^2}{d^2} Ad = \frac{1}{2}\left(\frac{\epsilon A}{d}\right) V^2$$

$$= \boxed{\frac{1}{2} CV^2}$$

d) $\text{energy} = 10V \cdot 10A \cdot h = 100\ W \cdot h$

$$100 W \cdot h \cdot \frac{3600\ s}{1h} = 3.6 \times 10^5\ J$$

$$C = \frac{\epsilon A}{d} = \frac{8.85 \times 10^{-12}}{10^{-6}} \cdot A = 8.85 \times 10^{-6} \cdot A$$

$$\text{energy} = \frac{1}{2} CV^2 = \frac{1}{2}(8.85 \times 10^{-6} \cdot A)(10 V)^2 = 3.6 \times 10^5 J$$

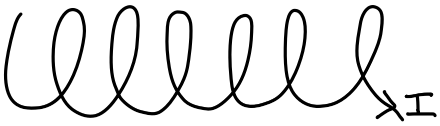
$$\rightarrow \boxed{A = 8.14 \times 10^8\ m^2}$$

Each plate has area $.1m \times .1m = .01m^2$

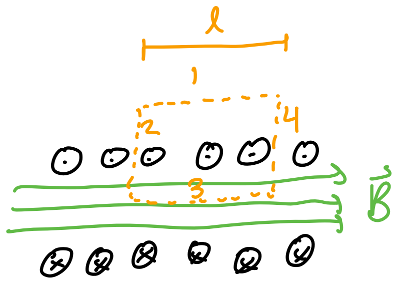
$$\frac{8.14 \times 10^8 m^2}{.01m^2} = 8.14 \times 10^{10} \text{ plates}$$

$$8.14 \times 10^{10} \text{ plates} \times 10^{-6} m \text{ thick} = \boxed{8.14 \times 10^4\ m} \approx 50 \text{ miles}$$

$\boxed{6.3}$ a)



n turns/m



$$\oint \vec{B}\cdot d\vec{s} = \int_1 \vec{B}\cdot d\vec{s} + \int_2 \vec{B}\cdot d\vec{s} + \int_3 \vec{B}\cdot d\vec{s} + \int_4 \vec{B}\cdot d\vec{s}$$

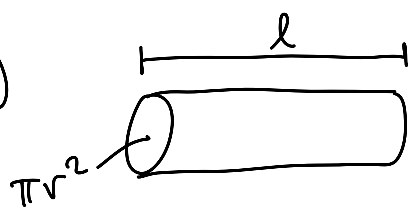
$$= 0 + 0 + B\ell + 0$$

Ampere's law: $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{enclosed}$

$\hookrightarrow$ $B\ell = \mu_0 \cdot N \cdot I$

$$B = \mu_0 I \frac{N}{\ell} = \boxed{\mu_0 I n}$$

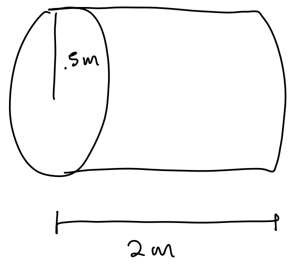
b)



$$u_m = \frac{B^2}{2\mu_0} = \frac{(\mu_0 I n)^2}{2\mu_0} = \frac{1}{2}\mu_0 I^2 n^2$$

$$U = \int_V u_m \, dV = \boxed{\frac{1}{2}\mu_0 I^2 n^2 \cdot \ell \cdot \pi r^2}$$

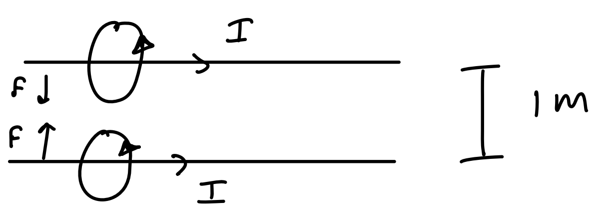
c)



$$\vec{f} = -\hat{r}\,\frac{d}{dr}W(r)$$

?

$\boxed{6.4}$ a)



Magnetic field of a current-carrying wire:

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{(Ampere's law)}$$

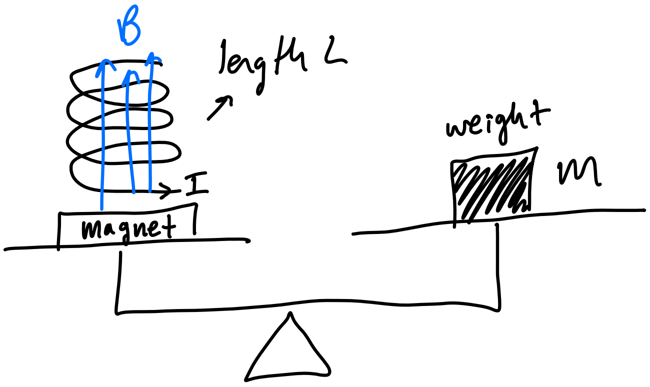
$$\vec{F} = \vec{I}L \times \vec{B} \rightarrow \frac{F}{\Delta L} = I \cdot B = \frac{\mu_0 I^2}{2\pi r}$$

$$2 \times 10^{-7}\ N/m = \frac{\mu_0 I^2}{2\pi \cdot 1m}$$

$\hookrightarrow I = 1\ A$ ✓

b) Problem: it depends on μ_0 definition

6.5



a) $\vec{F} = \vec{I} L \times \vec{B}$

$F = ILB = mg$

b) Faraday's law of induction:

$\mathcal{E} = V = -\frac{\Delta\Phi}{\Delta t} = BLv$

c) $BL = \frac{mg}{I} = \frac{V}{v}$

$\hookrightarrow m = \frac{IV}{vg}$

d) To allow BL to cancel out due to two seperate effects

6.6

a) $P_0 = \frac{E^2}{Z_0} = \frac{E^2}{377\Omega} = \frac{1000\ W}{m^2}$

$\hookrightarrow E = \boxed{614\ V/m}$

b) $\frac{1\ W}{1\ mm^2} \cdot \left(\frac{1000\ mm}{1\ m}\right)^2 = 10^6\ W/m^2$

$E = \sqrt{377\Omega \cdot 10^6\ W/m^2} = \boxed{1.94 \times 10^4\ V/m}$

$\frac{1\ W}{1\ \mu m^2} \cdot \left(\frac{10^6\ \mu m}{1\ m}\right)^2 = 10^{12}\ W/m^2$

$E = \sqrt{377\Omega \cdot 10^{12}\ W/m^2} = \boxed{1.94 \times 10^7\ V/m}$