Noise in Physical System

1.1) a) Binomial distrib : $p_m(x) = \frac{m!}{(m-x)!\,x!} p^x (1-p)^{m-x}$

for large m ($m \to \infty$) we have the stirling approx: $m! = \sqrt{2\pi}\, m^{m+1/2} e^{-m}$

$$\lim_{m\to\infty} p(x) = \lim_{m\to\infty} \frac{\sqrt{2\pi}\, m^{m+1/2} e^{-m}}{x!\,\sqrt{2\pi}\,(m-x)^{m-x+1/2} e^{-(m-x)}} p^x (1-p)^{m-x}$$

$$= \lim_{m\to\infty} \frac{m^{m+1/2}}{x!\,(m-x)^{m-x+1/2}\, e^{x}} p^x (1-p)^{m-x} \qquad (1)$$

for large $m \to$ Avg number of event $N = mp \leftarrow$ cte $\Rightarrow p = \frac{N}{m}$

plug in (1)

$$= \lim_{m\to\infty} \frac{m^{m+1/2}}{x!\,(m-x)^{m-x+1/2}\, e^{x}} \left(\frac{N}{m}\right)^x \left(1-\frac{N}{m}\right)^{m-x}$$

$$= \frac{N^x}{x!\,e^x} \lim_{m\to\infty} \frac{m^{m+1/2-x}}{(m-x)^{m-x+1/2}} \frac{1}{\cancel{m^x}} \left(1-\frac{N}{m}\right)^{m-x}$$

$$(m-x)^{m-x+1/2} = m^{m-x+1/2}\left(1-\frac{x}{m}\right)^{m-x+1/2}$$

$$= \frac{N^x}{x!\,e^x} \lim_{m\to\infty} \frac{(1-N/m)^{m-x}}{(1-x/m)^{m-x+1/2}}$$

$$= \frac{N^x}{x!\,e^x} \lim_{m\to\infty} \frac{\underbrace{(1-N/m)^m}_{e^{-N}}\underbrace{(1-N/m)^{-x}}_{1}}{\underbrace{(1-x/m)^m}_{e^{-x}}\underbrace{(1-x/m)^{-x}}_{1}\underbrace{(1-x/m)^{1/2}}_{1}}$$

$$= \frac{N^x}{x!\,e^x} \frac{e^{-N}}{e^{-x}} = \frac{e^{-N} N^x}{x!}$$

$\longrightarrow$ Poisson distrib is a case of the binomial distrib when $m \to \infty$

b) $$\langle x(x-1)\cdots(x-m+1)\rangle = \sum_{x=0}^{\infty} x(x-1)\cdots(x-m+1)\, p(x)$$

?

$$\frac{x(x-1)\cdots(x-m+1)}{x!} = \frac{1}{(x-m)!}$$

$$= \sum_{x=0}^{\infty} x(x-1)\cdots(x-m+1) \frac{e^{-N} N^x}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-N} N^x}{(x-m)!}$$

$$N^x = N^{m+x-m}$$

$$= N^m \underbrace{\sum_{x=0}^{\infty} \frac{e^{-N} N^{(x-m)}}{(x-m)!}}_{=1}$$

$$= N^m$$

$$\langle \underbrace{x}_{} + \overbrace{m-1}^{=1} \rangle = N^m$$

$$\sigma^2 = \langle (x-\mu)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

c) $\langle x \rangle = N$ $\Rightarrow \frac{\sigma}{\langle x \rangle} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$

$\sigma = \sqrt{m}$

c) We want to derive $\frac{\sigma}{\langle x \rangle} = \frac{1}{\sqrt{N}}$

let $m = 1 \rightarrow \langle x(x-1)\cdots(x-m+1)\rangle = \langle x \rangle = N$

let $m = 2 \rightarrow \langle x(x-1)\rangle = \langle x^2 - x \rangle = N^2 \rightarrow \langle x^2 \rangle = N^2 + N$

variance

$$\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 - 2x\langle x \rangle + \langle x \rangle^2 \rangle = \langle x^2 \rangle - 2\langle x \rangle^2 + \langle x \rangle^2$$

$$= \langle x^2 \rangle - \langle x \rangle^2$$

$$= N^2 + N - N^2$$

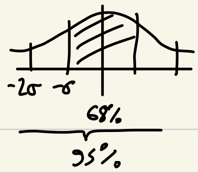
$$= N$$

$$\Rightarrow \sigma = \sqrt{N}$$

$$\Rightarrow \frac{\sigma}{\langle x \rangle} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

$\sigma \rightarrow$ std

$\sigma^2 \rightarrow$ variance



5.2) Rate /sec : N   large N

$\frac{\sigma}{\langle x \rangle} = \frac{1}{\sqrt{N}}$ → relative std in measuring N.

a) so we want $\frac{1}{\sqrt{N}} = 0.01 \Leftrightarrow N = 10^4$ At least 10^4 photon/sec to determine the photon count rate within 1%.

b) $\frac{1}{\sqrt{N}} = 1/1000\,000 \rightarrow N = 1.10^{12}$

c) $E = \frac{hc}{\lambda}$ watt = j/s

let's take $\lambda = 600$ nm → E in J per photon → to get in watt mult by N

$a \rightarrow \quad EN = \frac{Nhc}{\lambda} = 10^4 \; \frac{6{,}26 \cdot 10^{-34} \times 3 \cdot 10^{8}}{600 \cdot 10^{-9}} = 3{,}13 \cdot 10^{-15}$ watt

$b \rightarrow \quad = 10^{12}$ ——————— $= 3{,}13 \cdot 10^{-7}$ watt

5.3) Audio amplifier → 20 kHz Bandwidth.

a) source impedance : 10 kΩ

find V s.t SNR to Johnson noise = 20 dB

$$\langle V_{Noise}^2 \rangle = 4kTR\Delta f$$

$$= 4.(1{,}38 . 10^{-23}) . 300 . 10^4 . 2 . 10^4$$

$$\langle V_{Noise}^2 \rangle = 3{,}312 . 10^{-12} \quad V^2$$

$$SNR = 10 \log \left(\frac{\langle V_{signal}^2 \rangle}{\langle V_{Noise}^2 \rangle} \right)$$

$$\Rightarrow \quad 20 = 10 \log \left(\frac{\langle V_{signal}^2 \rangle}{3.3 * 10^{-12}} \right)$$

$$\Rightarrow \langle V_{signal}^2 \rangle = 3{,}3 . 10^{-10} \; V \quad \rightarrow \quad \underline{V_{signal} = 1{,}82 . 10^{-5} \; V}$$

b) Capacitor Energy $= \frac{1}{2} CV^2$?

c) Shot noise : $\langle I_{Noise}^2 \rangle = 2q\langle I \rangle \Delta f$

? Not sure

We want the noise to be within 1% of the current. $\frac{\sqrt{\langle I_{Noise} \rangle}}{\langle I \rangle} = 0.01$

$$\Rightarrow \sqrt{\frac{2q\Delta f}{\langle I \rangle}} = 0.01$$

$$\Rightarrow \langle I \rangle = \frac{2q\Delta f}{0.01^2} = 0{,}4 \times 10^{-11} \; A$$

5.4) ?