Semiconductor Materials and Devices Problem Set

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13.1a) Equation 12.27: $\left[\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}+V\right]\psi=E\psi$

$$\left[E+\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\right]\psi(x)=\sum_{n=\infty}^{\infty}V_0\,\delta(x-n\Delta)\psi(x)$$

Integrate from $x=-\epsilon$ to ϵ & take the limit $\epsilon\to 0$

$$\underbrace{\int_0^{\epsilon}E\,\psi(\hbar)m^2}_{\text{As }\epsilon\to 0,\text{ this}\to 0}=\underbrace{\int_0^{\epsilon}\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2}\,dx}_{\frac{-\hbar^2}{2m}\left(\frac{\delta\psi}{dn}\Big|_{\epsilon}-\frac{\delta\psi}{dn}\Big|_{0}\right)}+\underbrace{\int_0^{\epsilon}\Sigma V_0\,\delta(x-n\Delta)\psi(x)dx}_{V_0\psi(0)}$$

$$\frac{\hbar^2}{2m}\left(\psi(0^+)-\psi(0^-)\right)=V_0\,\psi(0)$$

Knowing that $\psi(x)=Ae^{iqx}+Be^{-iqx}$:

$$\frac{\hbar^2}{2m}\frac{d}{dx}\left(Ae^{i(qn-k\Delta)}+Be^{-i(qn+k\Delta)}-Ae^{i(qn)}-Be^{-i(qn)}\right)$$

Equation 12.28: $\frac{\hbar^2}{2m}iq\left(A-B-Ae^{i(q-k)\Delta}+Be^{-i(q+k)\Delta}\right)=V_0(A+B)$

13.1b) Using Bloch's Theorem (Equation 12.26) requiring $u_k(0)=u_k(\Delta)$

$$A+B=Ae^{i(q-k)\Delta}+Be^{-i(q+k)\Delta}$$

$$B=\frac{A}{1-e^{i(q-k)}}=\frac{A(1-e^{i(q-k)})}{e^{-i(q+k)\Delta}-1}\rightarrow \text{Plug in for } B \text{ and simplify.}$$

Since $e^{ix}=\cos x+i\sin x$: $\left(\cos(k\Delta)-\cos(q\Delta)-\frac{mV_0}{q\hbar^2}\sin(q\Delta)\right)A=0$

Equation 12.29: $\cos(k\Delta)=\cos(q\Delta)+\frac{mV_0\Delta}{\hbar^2}\frac{\sin(q\Delta)}{q\Delta}$

13.2) Equation 12.35: $f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$ (Fermi-Dirac Distribution)

For intrinsic (i.e., undoped) semiconductors, $E_F = \frac{E_c + E_v}{2} = E_c - \frac{E_g}{2}$

$\therefore f(E_c) = \frac{1}{1+e^{(E_c - E_F)/kT}} = \frac{1}{1+e^{E_g/2kT}}$

p.189

Plug in bandgaps: Ge is 0.67eV, Si is 1.11eV, diamond is 5eV

$$\text{Ge}: f(E_c) = \frac{1}{1+e^{0.67/(2\cdot 8.617\cdot 10^{-5}\cdot 300)}} = 2.35\cdot 10^{-6}$$

$$\text{Si}: f(E_c) = \frac{1}{1+e^{1.11/(2\cdot 8.617\cdot 10^{-5}\cdot 300)}} = 4.80\cdot 10^{-10}$$

$$\text{diamond}: f(E_c) = \frac{1}{1+e^{5/(2\cdot 8.617\cdot 10^{-5}\cdot 300)}} = 7.20\cdot 10^{-47}$$

expected occupancies

13.3a) $n_o p_o = n_i^2$

n-type Si $\therefore$ $n_o \approx N_D = 10^{17}\ cm^{-3}$ (for full ionization)

Plugging in: $p_o = \frac{n_i^2}{n_o} = \frac{(1.5\cdot 10^{10})^2}{10^{17}} = \frac{2.25\cdot 10^{20}}{10^{17}} = 2.25\cdot 10^{3}\ cm^{-3}$

13.3b) $E_F - E_i = kT \ln\left(\frac{n_o}{n_i}\right)$

Plugging in values: $E_F - E_i = 0.0259\cdot \ln\left(\frac{10^{17}}{1.5\cdot 10^{10}}\right) = 0.0259\cdot \ln(6.67\cdot 10^{6})$

Approximating $\ln(6.67\cdot 10^{6}) \approx \ln(10^{6}) + \ln(6.67)$

$E_F - E_i \approx 0.0259\cdot(13.82 + 1.90) = 0.407eV$

13.5a) $E_{stored} = \frac{1}{2}CV^2 = \frac{1}{2}(1\cdot10^{-15})(1.8)^2 = 1.62\cdot10^{-15}\,J$

13.5b) $E_{dissipated} = 1.62\cdot10^{-15}\,J$ since half the energy is stored and half is dissipated as heat during instantaneous capacitor charging through a resistor.

13.5c) For $\tau >> RC$, energy dissipated approaches zero. In other words, $\frac{RCV^2}{\tau}$ approaches 0 as τ approaches ∞.

13.5d) $E=CV^2$ (for each full cycle)

Plug in knowns: $E=CV^2 = 1\cdot10^{-15}\cdot(1.8)^2 = 3.24\cdot10^{-15}\,J$

To draw 1W: $f = \frac{1}{3.24\cdot10^{-15}} = 3.09\cdot10^{14}\,Hz$

13.5e) $E=CV^2 = 3.24\cdot10^{-15}\,J$

To find # events: 10^9 transistors $\cdot\, 10^9\,Hz = 10^{18}$ events/s

$P = 3.24\cdot10^{-15}\cdot10^{18} = 3.24\cdot10^{3}\,W = 3.24\,kW$

13.5f) $Q=CV$

Plug in knowns: $1\cdot10^{15}\cdot1.8 = 1.8\cdot10^{-15}\,C$

Since $e = 1.602\cdot10^{-19}\,C$: $n = \frac{Q}{e} = \frac{1.8\cdot10^{-15}}{1.602\cdot10^{-19}} = 1.12\cdot10^{4}$ electrons