PS4

Wednesday, March 5, 2025 8:54 PM

Q1)

$$A \times (B \times C) = B(A \cdot C) - C(A \cdot B)$$

$$(A \times (B \times C))_i = \epsilon_{ijk} A_j (B \times C)_k$$

$$(B \times C)_k = \epsilon_{klm} B_l C_m .$$

$$A \times (B \times C) = \varepsilon_{ijk} A_j \varepsilon_{klm} B_l C_m$$

$$\varepsilon_{ijk} \varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$$

$$\Rightarrow \; = (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) A_j B_l C_m .$$

$$= \delta_{il}\delta_{jm} A_j B_l C_m - \delta_{im}\delta_{jl} A_j B_l C_m$$

$$\delta_{jl} \quad j \rightleftarrows l \qquad \delta_{im} \quad i \leftarrow m$$

$$= A_m B_l C_m - A_l B_l C_i$$

$$\delta_{il} \quad \delta_{jm} \quad i \rightleftarrows l \quad j \rightleftarrows m$$

$$= B(A \cdot C) - C(A \cdot B).$$

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$$

$$\nabla(\nabla \cdot E) - \nabla^2 E$$

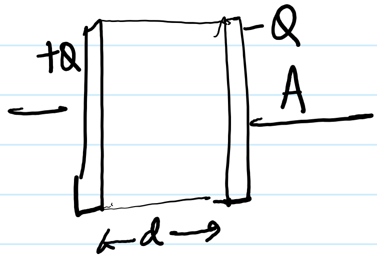
$\Leftarrow$ BAC CAB

$B \rightarrow \nabla$

$A \rightarrow \nabla \qquad C \leftrightarrow E$

Q2) a)

$$\int_V \nabla \cdot E \, dv = \oint_S E \cdot ds$$



$\Delta E = \rho / \epsilon$ since field is const.

$\leftrightarrow$ charge density

$$\rho = Q / Ad . \qquad \Delta E = \frac{Q}{A \epsilon d} .$$

$$\frac{Q}{A\epsilon d} \times Ad = E \times A \rightarrow E = Q/A\epsilon$$

$$V = \int^{d} E\,dl = \frac{Q}{A\epsilon} d$$

$$V = \frac{Q}{(\epsilon A/d)} = \frac{Q}{Z}$$

$(\epsilon A/d) \longrightarrow Z$ capacitance $\epsilon A/d$

b) $\nabla \cdot D = \rho$

$$\int \nabla \cdot D \, dv = \int \rho \, dv$$

$$\frac{d}{dt}\left(\int \nabla \cdot D \, dv\right) = \frac{d}{dt}(Q) = \rho$$

$$\frac{d}{dt}\int D \cdot dA \rightarrow \epsilon \frac{d}{dt}\left(\int E \cdot dA\right)$$

$$= \epsilon \frac{d}{dt}\left(\int \frac{V}{d} \, dA\right)$$

$$I_D = \epsilon \frac{dV}{dt} \frac{A}{d}$$

$$= C \frac{dV}{dt}$$

c) $U = \frac{1}{2}\epsilon E^2$ ($\frac{1}{2} DE$) $\rightarrow \frac{1}{2}\frac{CV^2}{Ad}$ $(E = V/d)$

$$E = \int U \, dV$$

$$= \int \frac{1}{2}\epsilon E^2 \, dV = \underline{\underline{\frac{1}{2}\epsilon A d E^2}} = \frac{1}{2}\frac{\epsilon A}{d} V^2$$

$$= \frac{1}{2} C V^2$$

d) $W_{ht} = \frac{1}{2} C V^2$

$U = 10 \times 36\,000\,C$

$Q \times V$

$U = 10 \times 36\,000\,C$

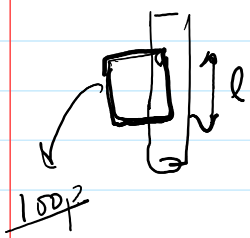
$\underline{Q \times V}$

$$A = \frac{2Ud}{\epsilon V^2} \approx \underline{8.1\times10^{8}}\ m^2$$

Q3)

a) $\oint^{C} H \cdot dl = \int^{S} \left(\cancel{\frac{dD}{dt}} + J\right)\cdot dA$

$\frac{dD}{dt} \to 0$ in static magnetic field.



$$\oint^{C} H \cdot dl = \int^{S} J \cdot dA$$

$= H\times l + 0 \quad = n \times I \times l \qquad n \to$ turns/unit l.

$$H = \frac{nIl}{l} = \underline{nI}$$

$$B = \mu_0 H = \underline{\mu_0 n I}$$

b) $U = \frac{1}{2}\frac{B^2}{\mu_0}$

$$U_{total} = \frac{1}{2}\frac{(\mu_0 n I)^2}{\mu_0} \times \pi r^2 l.$$

$$= \underline{\pi \mu_0 n^2 I^2 r^2 l}$$

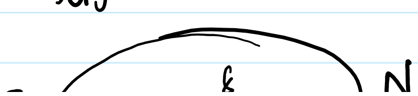
c)

$$U_{tot} = \frac{1}{2}\frac{B^2}{\mu_0}\pi r^2 l.$$

$$= 6.25 \times 10^{7}\ J.$$

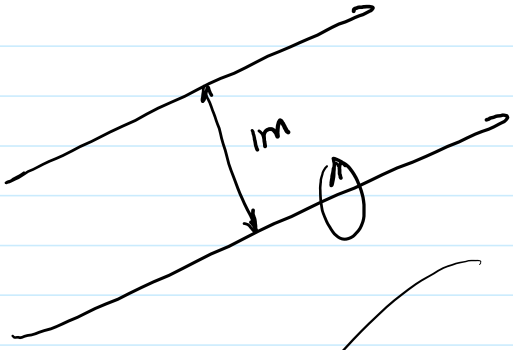
$B = 10\ T$
$l = 5\ m$
$r = 0.5\ m.$

$$F = \frac{\delta U}{\delta r} = \frac{B^2}{\mu_0} 2\pi r l$$



$$= \boxed{2.5\times10^{8}}\ \text{N}$$

Q4)



$$F = 2\times10^{7}\ \text{N}$$

$$dF = I\,\vec{dl}\times\vec{B}$$

$$\oint B\cdot dl = \mu_0 I$$

$$2\pi R B = \mu_0 I$$

$$I = \frac{2\pi R B}{\mu_0}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

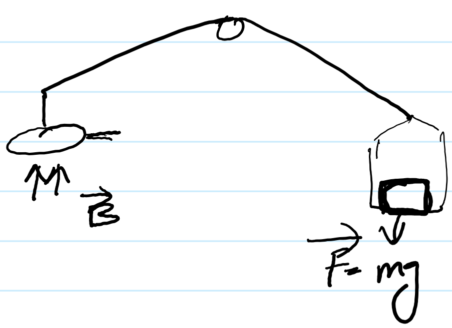
$$F = ILB$$

$$\frac{F}{L} = IB = \frac{2\pi R I B}{\mu_0}$$

$$I = 1\text{A}$$

$$F/L = \underline{\underline{2\times10^{-7}\ \text{N/m}}}$$

5) Kibble balance.



$$dF = I\,\vec{dl}\times\vec{B}$$

$$\int dF = F = I\int dl\times B \qquad (1)$$

$$F_z = I\int\left(B_y\,dx - B_x\,dy\right)$$

Greens Theorem

$$= I\int \frac{\delta B}{\delta z}\,dA$$

$$= \underline{\underline{IA\frac{\delta B}{\delta z}}}$$

$$V = -\oint \vec{E}\cdot\vec{dl}$$

$$= -\int(\nabla\times E)\,dA \qquad (2)$$

$$= \int N\frac{\delta B_z}{\delta t}\cdot dA$$

$$= \int N \frac{\delta B_z}{\delta t} \cdot dA$$

$$\frac{}{\delta t}$$

$$\Rightarrow \frac{\delta B_z}{\delta t} A = \frac{V}{N} = \frac{f_z}{I}$$

③

$$f_z = \frac{VI}{N} = mg$$

$$\Rightarrow m = \frac{VI}{Ng}$$

6) $1 kW/m^2$

$$\vec{P} = \vec{E} \times \vec{H}$$

$$E = -\sqrt{\frac{\mu_0}{\epsilon_0}} H$$

$$\Rightarrow P = \sqrt{\frac{\epsilon_0}{\mu_0}} E^2$$

$$E = E_0 e^{i(kx - \omega t)}$$

$$|e^{ikx}| = 1$$

$$Re(e^{i\omega t})^2 = \cos(\omega t)^2$$

$$avg = \frac{1}{2}.$$

$$P = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}} |E|^2$$

$$10^3 = P/Area.$$

$$\therefore E_0 \approx 1 kV \longrightarrow$$ Dielectric breakdown.

Causes sunburn